On Properties and Applications of A New Form of Discrete Time Optimal Control Law

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Abstract: Unlike the well-known bang-bang control for continuous plants, the closed-form time optimal control for discrete time plants was obtained only recently [1,11]. It is shown in this paper that this new discrete time optimal control (DTOC) law offers a practical alternative to bang-bang control. A particularly interesting property of DTOC is that the control signal is not always bang-bang. Instead of constant chattering, DTOC can produce a smooth control signal that results in a similar performance to that of the bang-bang control. Helpful insight is offered on how to maintain the smoothness of control signal in the presence of significant sensor noises. Superior robustness of the new control law is demonstrated for a general second order system with substantial dynamic uncertainties. Implementation issues and the tuning of the control parameters are also discussed. Finally, the performance of the new control law is examined in an industrial motion control case study and a computer hard disk drive problem.

Keywords: Time Optimal Control, Bang-Bang Control, Discrete Time Optimal Control, Motion Control.

I. Introduction

Servo control problems are one of the most common type of problems in manufacturing, as well as the defense industry. The study of this problem, started in the 1950s, led to the formulation of Time Optimal Control (TOC) problem and its solution, the Bang-Bang controller [2-4,9,10]. The research also led to the developments of Optimal Control theory [6-8].

The TOC formulation captures the essence of many practical control problems, i.e., to reach the setpoint in the shortest time possible with limited actuator ranges. The application of its solution, Bang-Bang control, however, is quite limited because it unavoidably leads to undesirable control signal chattering, which could, in turn, lead to excessive wear and tear of the actuators.

The work by J. Han and L. Yuan [11] provides an alternative mathematical solution to the DTOC problem. It was derived, using the isochronic regions (IR), for a discrete time, double-integral system. A closed-form solution is obtained and it demonstrates that the solution for DTOC problem is not necessarily bang-bang control. In fact, as shown below, the new solution completely resolved the chattering issue without compromising the performance. Detailed derivations of the control law can be found in [1].

To summarize the new DTOC results, consider the discrete double integral plant

$$x(k+1) = Ax(k) + Bu(k)\,,\ |u(k)| \le r \tag{1.1}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, A= $\begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 0 \\ h \end{bmatrix}$

Note that h is the sampling period and r is the actuator saturation limit. The DTOC problem is defined as follows.

Definition 1: Given the plant (1.1) and its initial state *x(0)*, determining the control signal sequence, *u(0), u(1), …, u(k)*, such that the state *x(k)* is driven back to the origin in a minimum number of steps, subject to the constraint of $|u(k)| \le r$, i.e.,

find $u(k^*)$, $|u(k)| \le r$, such that $k^* = \min\{k | x(k) = 0\}$ (1.2)

The solution of this DTOC problem proposed in [1,11] is

$$u = -r sat(a(x_1, x_2, r, h), hr) \tag{1.3}$$

where

$$a(x_1, x_2, r, h) = \begin{cases} x_2 + \dfrac{\sqrt{h^2r^2 + 8r|y|} - hr}{2} sign(y), & |y| > h^2 r \\ x_2 + y/h, & |y| \le h^2 r \end{cases}$$

$y = x_1 + hx_2$

This control law can be easily implemented in a digital computer as

$$\begin{array}{|l|} \hline u = fst((x_1, x_2, r, h) \\ d = rh;\ d_0 = hd \\ y = x_1 + hx_2 \\ a_0 = \sqrt{d^2 + 8r|y|} \\ a = \begin{cases} x_2 + \dfrac{a_0 - d}{2} sign(y), & |y| > d_0 \\ x_2 + y/h, & |y| \le d_0 \end{cases} \\ fst = -\begin{cases} r\ sign(a), & |a| > d \\ r\dfrac{a}{d}, & |a| \le d \end{cases} \\ \hline \end{array} \tag{1.4}$$

Note that the well-known TOC solution for a continuous double integral plant, also known as the bang-bang control [2-10] is

$$u = -r\ sign(x_1 + \frac{x_2|x_2|}{2r}) \tag{1.5}$$

The penalty associated with this control law is the frequent switching of the control signal between its two extreme values around the switching curve described in

(1.5), particularly around the origin, $x=0$. Furthermore, the instant switching requires an infinitely large $\dot{u}$, which is usually not practical. Many modifications of the control law (1.5) were made to ease the implementation:

1) Adding a dead zone:

$$u = 0, if \|x\| < \delta \qquad (1.6)$$

can be added to (1.5) to reduce the chattering of u around the origin. δ can be chosen by trial and error or set to

$$\delta = \|n\|_\infty \qquad (1.7)$$

where n is the measurement noise in x. Or,

2) Adding a linear region:

$$u = k_1 x_1 + k_2 x_2, if \|x\| < \delta \qquad (1.8)$$

where k_1 and k_2 are linear gains to be selected.

3) Adding a saturation zone:

$$u = -r \ \mathrm{sat}(x_1 + \frac{x_2 |x_2|}{2r}, \delta) \qquad (1.9)$$

where the saturation function is defined as

$$sat(s,\delta) = \begin{cases} sign(s) & s > \delta \\ \dfrac{s}{\delta} & |s| \le \delta \end{cases} \qquad (1.10)$$

These modifications are rather ad-hoc, representing the trade-off between the speed of convergence and the smoothness of the control signal. As demonstrated in this paper, the new DTOC solution fundamentally resolved this conflict and provided a practical time optimal control solution.

The paper is organized as follows. The properties of DTOC are discussed in section II. The implementation issues are examined in section III. A case study in a motion control problem is presented in section IV and further illustrated in section V on a computer hard disk drive control problem. Finally the concluding remarks can be found in section VI.

II. Properties of DTOC

A common goal in practical control design is to achieve maximum closed-loop bandwidth subject to physical and stability constraints. This is because the higher the bandwidth, the better the command following and the disturbance rejection. In motion control applications, "motion profile" is often used to provide the desired transient response. The design goal is to make the actual system follow this profile as quickly and closely as possible. The quality of the following can be indeed measured by the closed-loop bandwidth, which directly impacts the rise and settling time. That is, the maximum bandwidth results in minimum transient time. For example, in a computer hard-disk drive, which is one of the most challenging motion control problems, the performance of the controllers are compared using the bandwidth because of its straight-forward relationship to the critical specification: the seek time.

The TOC problem, as defined above, is similar to the maximum bandwidth problem. The key difference, however, is that this is a nonlinear solution. It is demonstrated in this paper that DTOC provides excellent solution for robust control and is extremely simple in implementation and tuning. More importantly, the DTOC solution provides a powerful tool for engineers to seek compromise in performance and smoothness, and it has great potential for industrial applications.

2.1. Performance Comparisons

The key difference between the conventional continuous time optimal control (CTOC) and the DTOC solutions is that the control signal of DTOC is inherently smooth. Figure 2.1 shows the simulation setup of the comparison test, where the reference signal is a unit step function. Here, DTOC (r=5, h=.002), with the sampling period of T_s=.001 is compared to CTOC (r=5), as defined in [1], equation (2.6), and to the modified CTOC (MCTOC) defined in [1], equation (2.9), where r =5, δ=.01, and k_1=k_2=1. The simulation is carried out in Simulink using the ode45 numerical integration algorithm with variable step size. The tracking errors and the control signals are captured and shown in Figure 2.2.

From the simulation results in Figure 2.2 one may make the following observations:

- In comparison, there are two problems in CTOC: 1) The control signal constantly bounces between the two extreme values; 2) there is an overshoot as the output approaches the steady state, i.e., it is not truly time optimal even though the simulation employs a good numerical integration algorithm.
- The MCTOC is successful in eliminating the high speed control signal chattering at the cost of less desirable command following performance (steady state error, slower transient response).

The DTOC is the obvious winner with the fastest converging time, no overshoot and smoother control signal. It supports the mathematical derivation that the time optimal control for discrete time system is not always a bang-bang control.

2.2 Robustness and Invariance of DTOC

For a control method to be practical, it must be robust, which means the control system should be able to withstand the dynamic uncertainties in the plant and external disturbances. How much uncertainty and disturbance it can tolerate while keeping the performance within the specifications offers a basis for comparison of various controllers.

DTOC was derived for the double-integral plant defined in (1.1), which is the discrete time state space form of

$$\ddot{y} = u \ , \ |u| \le r \qquad (2.1)$$

Now consider a second order system

$$\ddot{y} = f(t, y, \dot{y}, w) + u \; , \; |u| \le r \qquad (2.2)$$

where y is the output, u the input and w the disturbance. The conventional way of solving this problem is to identify, or estimate, $f(t, y, \dot{y}, w)$, which yields a mathematical form from which a control law is designed. Here we offer a different perspective: let's assume that $f(t, y, \dot{y}, w)$ is totally unknown and, to us, the control problem is that of a double-integral problem. Let's see if the DTOC law is powerful enough to compensate for $f(t, y, \dot{y}, w)$, without the knowledge of it. To make the task challenging, let's use a pulse sequence in place of $f(t, y, \dot{y}, w)$ in simulation with a magnitude of ± 50% of r. The simulation setup is shown in Figure 2.3. Simulation results in Figure 2.4 show that the DTOC was able to compensate for the particular, pulse-like, $f(t, y, \dot{y}, w)$ quickly and efficiently.

Robustness of DTOC:

The level of robustness shown in Figure 2.3 and 2.4 is rarely seen in existing methods. The robustness and disturbance rejection can be viewed, in this context, as one problem: to overcome $f(t, y, \dot{y}, w)$. Philosophically, rather than relying on the prior knowledge of the plant, i.e., $f(t, y, \dot{y}, w)$, DTOC reacts aggressively to slight deviation in y and $\dot{y}$ from the setpoint at each sampling instant. This begs the question of what we really need to know in order to control a plant, its model in terms of $f(t, y, \dot{y}, w)$ that describes the global dynamics during the operation, or its local behavior of e, $\dot{e}$ ... at each sampling instant. This example suggests that DTOC is a suitable solution for a class of control problems, and that the local behavior based design philosophy could very well be the design of choice, practically speaking. More in-depth analysis is needed to verify this claim and will be discussed in future publications.

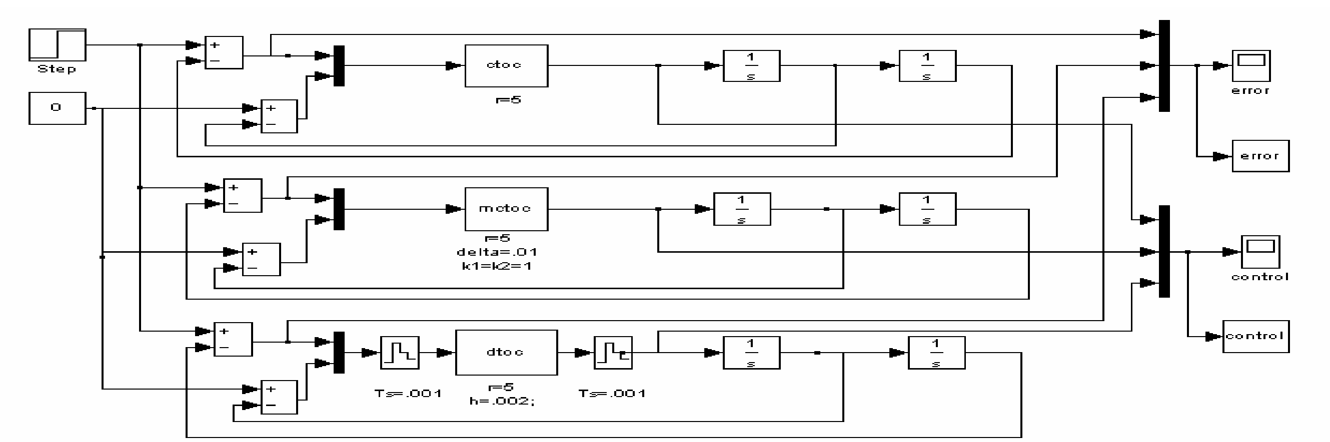


Figure 2.1 Simulation Setup

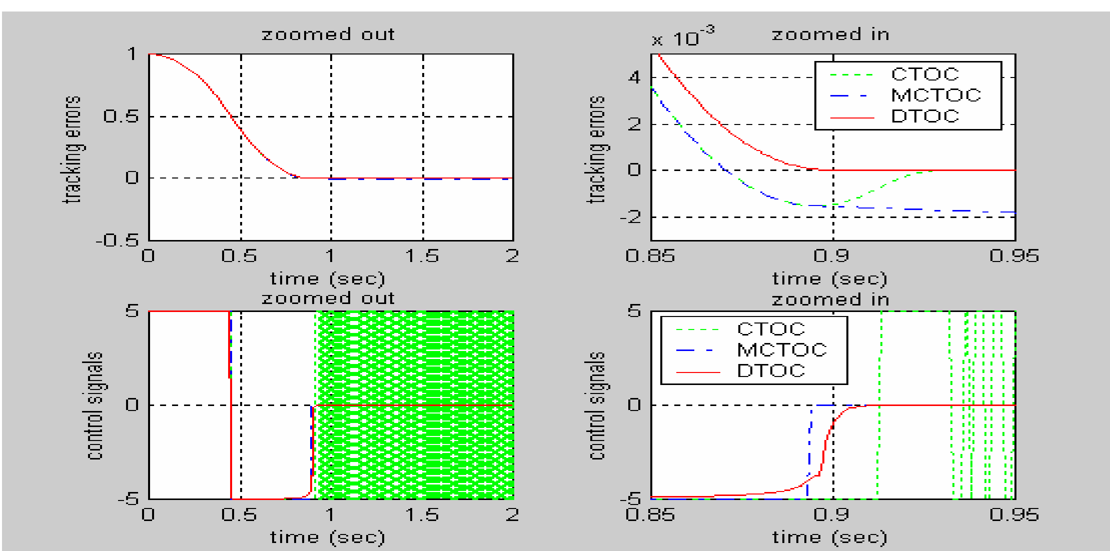


Figure 2.2 Comparisons of CTOC, MCTOC and DTOC

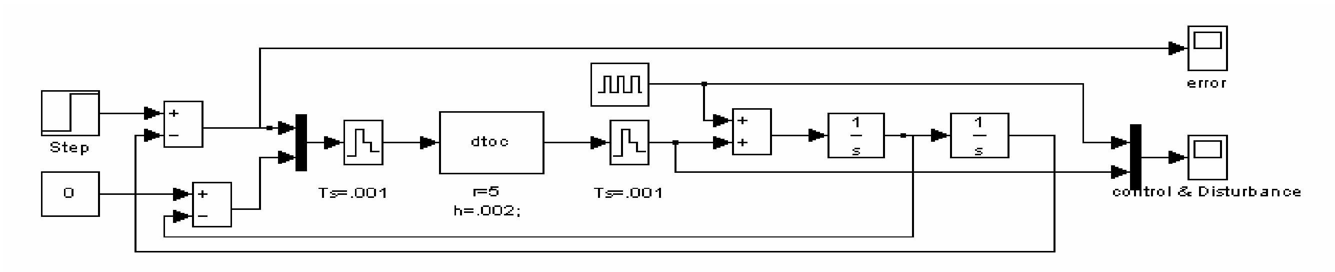


Figure 2.3 Simulation setup for robustness test

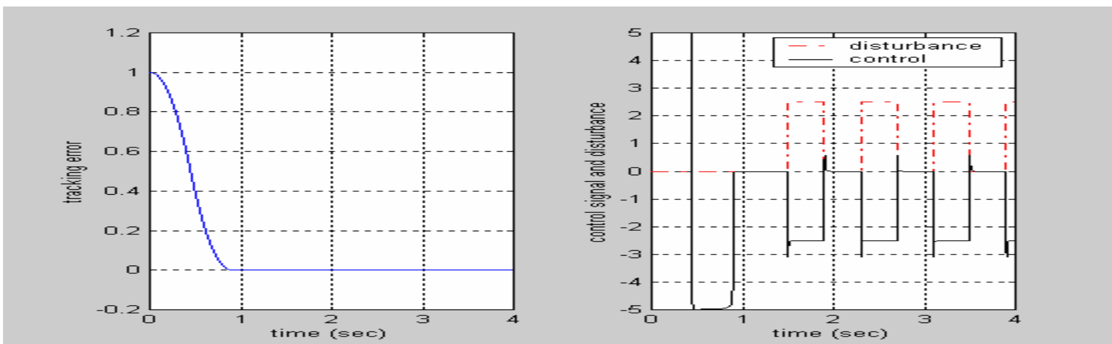


Figure 2.4 Robustness of DTOC

III. Implementation Issues

DTOC has exhibited many desirable properties. This section addresses the limitations of DTOC in practical applications.

3.1 Extending DTOC to general second-order plants

First, let's consider a double-integral plant with non-unity gain:

$$\ddot{y} = bu\ ,\ |u| \le r \tag{3.1}$$

Let $v=bu$, (3.1) is equivalent to the problem of

$$\ddot{y} = v,\quad |v| \le br \tag{3.2}$$

Then the control law solution of (2.3) can be derived from that of (2.4) by

$$u=v/b \tag{3.3}$$

Therefore, the DTOC law of (1.3) for the plant (3.1) is

$$u= -r\ sat(a(x_1, x_2, br, h),\ hbr) \tag{3.4}$$

This control law can be implemented in a digital computer as

$$u = (1/b) fst((x_1, x_2, br, h) \tag{3.5}$$

where the *fst(.)* function is defined in (1.4).

It was demonstrated above that the DTOC has good disturbance rejection for the second order plant $\ddot{y} = f(y, \dot{y}, w, t) + u\ ,\ |u| \le r$. The above derivation now extends the scope of DTOC to

$$\ddot{y} = f(y, \dot{y}, w, t) + bu\ ,\ |u| \le r \tag{3.6}$$

which is a general second order nonlinear plant. Here $f(y, \dot{y}, w, t)$ represents the unknown plant dynamics and external disturbance and b is the only known parameter in the system. In the case where b is not known, its estimate, $b_0 \approx b$, is needed and (3.6) can be rewritten as

$$\ddot{y} = [f(y, \dot{y}, w, t) + (b - b_0)u] + b_0 u\ ,\ |u| \le r \tag{3.7}$$

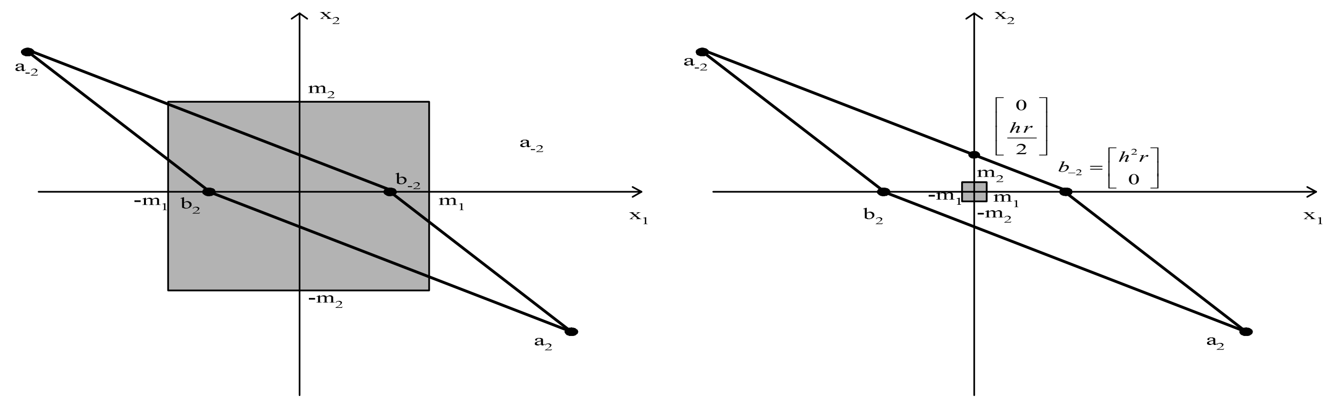
Clearly, the estimation error can be dealt with as part of the unknown disturbance.

3.2 Noise and bandwidth issues

Like any control design method, DTOC has advantages and disadvantages. By nature, time optimal control is a very aggressive design. It is similar to a linear controller designed for a maximum bandwidth because both of them seek to reduce the tracking error in the shortest time permissible. Since the controller performance is usually measured in how well the output of the plant tracks the reference, especially in the presence of disturbances, the design objective of time optimal control seems to fit the engineering applications quite well.

On the other hand, TOC designs have several practical problems. First and foremost is the high-speed chattering in the control signal that often results in excessive wear and tear on control actuators. The DTOC algorithm seems to have resolved this problem. Another issue is that the achievable bandwidth is limited by the noise level in the measurement and by dynamic uncertainties in the plant, such as the resonant modes. In practice, the choice of bandwidth is usually a result of compromise made among several competing design goals, including:

- performance (command following and disturbance rejection);
- control signal smoothness; and
- stability in the presence of uncertainties.



(a) False Switching Scenario (b)Expanded Switching Surface

Figure 3.1 Switching surface and uncertainties created by sensor noises

A TOC type of controller is especially vulnerable to noises because it requires both the output ($x_1= y$) and its differentiation ($x_2= \dot{y}$) information and x_2 is usually not available directly. The estimated $\dot{y}$ from the measurement of y is very sensitive to noises in y. Because of the high bandwidth design, using a low pass filter to attenuate the noises in y unavoidably degrades the performance of the controller. This noise issue is common in all high bandwidth controller designs, and DTOC is without exception.

To understand this limitation better, assume that the inputs to DTOC are

$$\begin{aligned}\overline{x}_1 &= x_1 + n_1,\ \ |n_1| < m_1\\ \overline{x}_2 &= x_2 + n_2,\ \ |n_2| < m_2\end{aligned} \tag{3.8}$$

where $\overline{x}_1$ and $\overline{x}_2$ are noise-corrupted version of x_1 and x_2, respectively. n_1 and n_2 are noises defined as zero-mean random numbers, bounded by m_1 and m_2, respectively. With $x=[x_1, x_2]^T=[0\ 0]^T$ at the origin, (3.8) defines an area on the phase plane, as shown in Figure 3.1 as the shaded rectangle.

Note that the parallelogram in Figure 3.1 represents the switching surface corresponding to the control law of (1.4). For any initial state inside it, (1.4) forces the state back to the origin within two sample steps. Figure 3.1a describes a scenario where the noise is too large for the DTOC to be effective. It frequently leads to false switching and it makes the control signal unsmooth. The shaded area in Figure 3.1b, represents an acceptable uncertainty in x that is well contained within the switching surface, where the noise-induced uncertainty amounts to a small part. Therefore, the control signal will not make a sudden big change due to noises.

<u>Design Trade-offs</u>

In a linear control design, a common approach is to reduce the loop bandwidth in the face of uncertainties in the measurement and in the dynamics. The reasoning can be rigorously established through the frequency response of the loop gain, which is the basis of the loop-shaping design. For DTOC, the trade-offs between its aggressiveness and the smoothness of the control signal can be made as follows:

1) Using a low pass filter to reduce noises in x;
2) Enlarge the regions in (1.4) so that the noises do not cause the bang-bang control action;
3) A combination of 1) and 2).

The first option is a common one where low pass filters are used to filter out part of the high frequency noises. The main drawback of this is the additional phase lag that accompanies the low pass filters, which limits the performance and reduces the stability margins. Note that the parallelogram in Figure 3.1 is defined by the four points:

$$\{a_2=\begin{bmatrix}3h^2r\\ -2hr\end{bmatrix},\ a_{-2}=\begin{bmatrix}-3h^2r\\ 2hr\end{bmatrix},\ b_2=\begin{bmatrix}-h^2r\\ 0\end{bmatrix},\ b_{-2}=\begin{bmatrix}h^2r\\ 0\end{bmatrix}\}$$

If h and r are treated as controller parameters to be tuned, then the region can be enlarged by increasing either or both parameters. Let

$$\begin{aligned}\overline{r} &= k_r r\\ \overline{h} &= k_h h\end{aligned} \tag{3.9}$$

and replace r and h in DTOC law in (3.5) with $\overline{r}$ and $\overline{h}$, respectively, we have

$$u = \frac{1}{bk_r} fst((x_1, x_2, b\overline{r}, \overline{h}) \tag{3.10}$$

Here k_r and k_h are considered filter coefficients. It was observed in practice that setting k_r=1 and adjusting k_h as the only tuning parameter is a simple and effective tuning method.

IV. A Motion Control Case Study

Consider a motion control test bed as shown in Figure 4.1. The mathematical model of the motion system was derived and verified in hardware test, as

$$\ddot{y} = (-1.41\dot{y} + 23.2T_d) + 23.2u \tag{4.1}$$

where y is the output position, u is the voltage signal sent to the power amplifier that drives the motor, and T_d is the torque disturbance. The design objective is to rotate the load one revolution in one second with no overshoot. A trapezoidal motion profile is used to provide the desired trajectory for the output of the plant to follow. Here the physical characteristics of this control problem are 1) $|u|<3.5$ volt, 2) the sampling rate is 1 kHz, 3) there could be a torque disturbance up to 10% of the maximum torque, 4) the noise level in the control signal should be within ±100mV in the presence of sensor noise.

Figure 4.1 The Motion Control Plant

In motion control applications, the common solution is a linear proportional-derivative (LPD) controller of the form

$$u = k_p(r - y) + k_d(-\dot{y})$$

Assuming the mathematical model of the plant is known, i.e., the parameters of (2.13) are given, the LPD gains can be selected as [17]

$$k_p = .086\omega_c^2 \text{ and } k_d = .061(2\omega_c - 1)$$

which results in a closed-loop transfer function of

$$G_{cl}(s) = \frac{\omega_c^2}{(s+\omega_c)^2}$$

Here,

ω_c =60 rad/sec

is selected as the maximum bandwidth achievable, subject to the condition that noise level in the control signal is within 100 mV. Furthermore, to avoid noise corruption of the control signal, an approximate differentiator $\frac{s}{(\tau s+1)^2}$ is applied with a corner frequency of $10\omega_c$ selected so that the approximation of the differentiator does not introduce problematic phase delays at the crossover frequency. That is $\tau=1/(10\omega_c)$. The approximate differentiator is used for both the PD and the DTOC controllers. To make the comparison fair, the DTOC parameters are selected so that the noise level in the control signal is the same as that of the LPD controller. Here, the DTOC parameters are selected as k_r=10, k_h=2, τ=.001. The performance of both controllers is demonstrated in Figure 4.2. The desired trajectory of the output is given based on a trapezoidal motion profile. A sinusoidal torque disturbance of 2Hz is introduced at t=1.5 sec.

The simulation results show that the tracking error of DTOC is over a hundred times smaller than that of LPD during the transient period and in disturbance rejection where the error is more than three times smaller.

Remarks

DTOC, in a way, is a nonlinear proportional-derivative controller (NPD). This comparison is fair because both controllers are “optimal”. DTOC is optimal, of course, inherently, and the LPD is optimal in the sense that its bandwidth is maximized subject to design constraints. They share the disadvantage of any PD controller in terms of requiring the differentiation of the output signal and are therefore sensitive to output noises. Another common problem with a PD controller is its difficulty in eliminating the steady state errors completely in the presence of disturbances. This steady error is not obvious when the sensor noise level is low and the controller can be designed aggressively. The above example shows that with the same sensor noise level, DTOC provides a better solution.

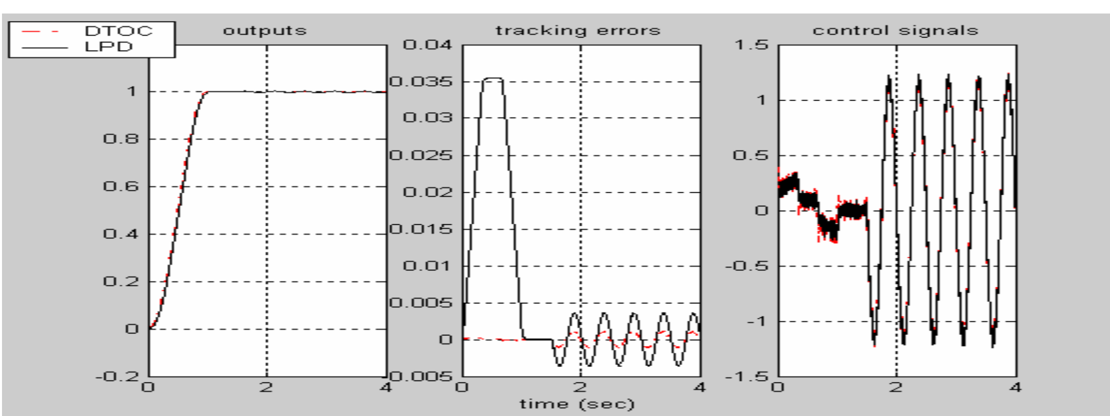


Figure 4.2 Comparison of DTOC and LPD in a motion control application

V. An Application to Computer Hard Disk Drive

The position control in a computer hard disk drive is one of the most challenging problems in motion control. Five characteristics set it apart from other applications: 1) high following accuracy; 2) high seek speed; 3) resonant modes; and 4) position and torque disturbances; 5) power amplifier output saturation. The first two are obvious and 3) is due to the fact that the hard disk drive assembly (HDA) is made light, therefore flexible, in order to move it fast. This creates over a hundred resonant modes but only a few of them affect the control performance significantly [18, 19]. The accuracy requirement also dictates that the motion control system has superior disturbance rejection capabilities.

The mathematical model for the HDA is approximately

$$\frac{K_v K_y}{s(s + K_{fric}/m)} H_d(s) \qquad (5.1)$$

where the output is the position, the input is the motor current, K_y is the position measurement gain, $K_v = K_t/m$ is the acceleration constant, K_{fric} is the viscous friction coefficient, and m is the moving mass of the actuator. $H_d(s)$ represents the dynamics from the resonant modes

$$H_d(s) = \sum_{j=1}^{8} \frac{\omega_j b_{2j} s + \omega_j^2 b_{2j-1}}{s^2 + 2\omega_j \xi_j s + \omega_j^2} \qquad (5.2)$$

where ω_j is the resonance frequency, ξ_j is the damping ratio, and b_j is the coupling coefficient.

In conventional HDD servo systems, Mode Switching Control (MSC) is mostly employed (see [18] and references therein), which consists of two different control schemes to achieve both short seek-time and satisfactory track-following performance. With the HDD evolving towards smaller size, larger volume and faster access speed, current HDDs using the combination of classical control techniques can no longer meet the increasing performance requirements.

A Time Optimal Unified Servo Controller (TOUSC) is proposed in [18] based on the DTOC concept described above. Ignoring the friction and resonant modes, the plant in (5.1) is treated as a double integrator and the control law (1.4) is applied. To estimate the x_2 in (1.1), the current estimator from [19] is applied. The system configuration is shown in Figure 5.1. Interested readers are referred to [18] for details on the plant and the controller.

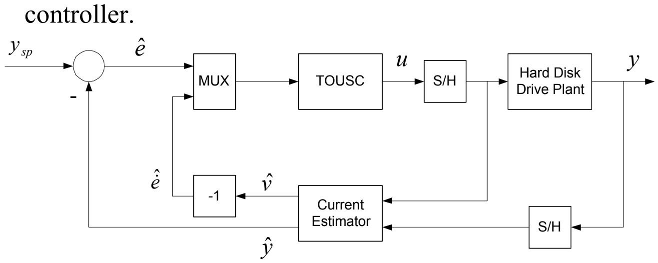


Figure 5.1 TOUSC Configuration

The simulation results based on a 13kTPI industrial HDD plant are shown in Figure 5.2 and 5.3, where the setpoint is 10,000 tracks.

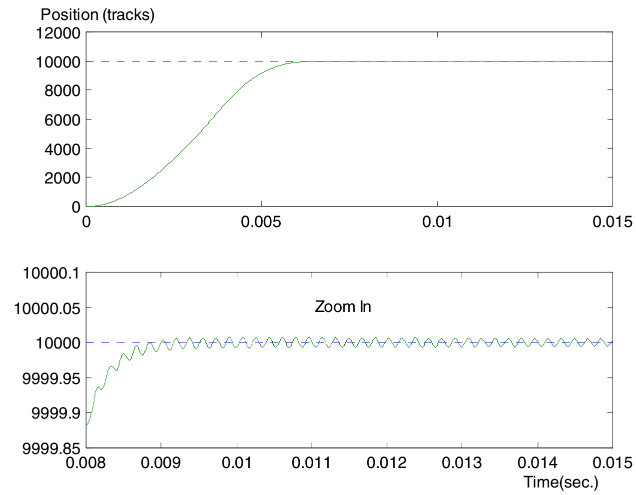


Figure 5.2 Position response of the TOUSC

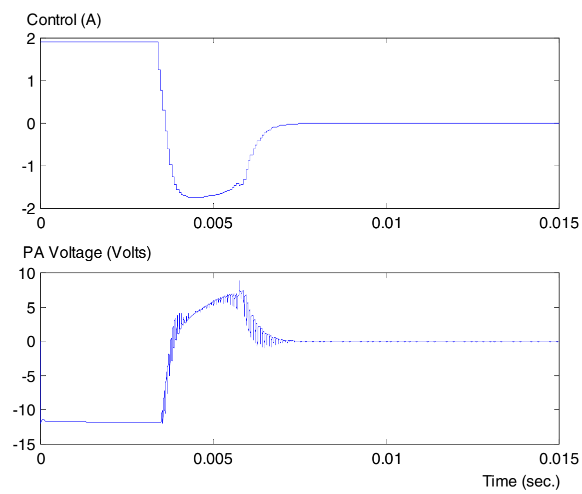


Figure 5.3 Motor current and Power Amplifier voltage

***Remarks*:**

There are several remarkable characteristics of TOUSC:

1) Only one controller is needed (which is why the new method is called unified) ;
2) The accuracy is extremely high (Figure 5.2)
3) There is no overshoot, as expected from a time optimal control law;
4) Smooth motor current (which means less wear and tear for the motor);
5) The control design is very simple; The controller is designed for a double integrator but the simulation was carried out with a full simulation model that includes friction and unmodeled dynamics (resonant modes).
6) The tuning is straightforward. With the observer (i.e the current estimator in Figure 5.1) properly setup,

there is only one tunable control parameter (k_h) here with k_r fixed at k_r=1.

7) The current estimator from [19] plays a crucial role here in providing an estimation of x_2 in (1.1) with minimum phase lag.

VI. Concluding Remarks

A new time optimal control law is evaluated in this paper for its performance and practicality. Design and tunings issues are addressed. Special attention is paid to practical issues such as noise handling and disturbance rejection. The advantages and disadvantages of this nonlinear proportional-derivative control law are demonstrated in motion control applications. This new time optimal control law resolves the long standing issue of chattering in the control signal and is, therefore, much more practical than the well-known bang-bang control solution.

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