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An active disturbance rejection control solution for hysteresis compensation

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***Abstract*—An Active Disturbance Rejection Control (ADRC) is proposed herein whereby hysteresis with unknown characteristics is treated as disturbance and rejected. In place of the prevailing model inversion method, which requires detailed mathematical model of the hysteresis, the proposed active disturbance rejection approach offers an appealing alternative to hysteresis compensation, one that does not require a detailed model of hysteresis. The benefits include the ease of initial design and the potential for continued operation without recalibration, as compared to the existing model based methods such as model inversion. Promising results are obtained via simulation in applying the proposed method to typical hysteresis compensation problems found in a multitude of processes and applications, which imposes severe limitations on their performance. Beyond the particular solution offered here, we believe that the readers will find in this paper a new way of thinking in regards to hysteresis compensation.**

I. Introduction

There exist today a multitude of processes and applications which exhibit hysteresis as their defining characteristic. Hysteresis may be beneficial in some processes where energy conservation is valued, yet there are many processes wherein the hysteresis characteristic imposes severe limitations meeting the desired operating bandwidth, balancing stability and precision with speed. The challenge of hysteresis compensation affects a diverse spectrum of applications, spanning the study of cell cycle mitosis [1], DNA coiling and extension [2], multilane vehicle traffic flow [3], assisted sperm injection [4] and medical robotics [5], to name a few.

The hysteretic application chosen for demonstration is precision positioning using a piezoceramic actuator. This application is extremely challenging for the fledgling nanotechnology industry and has generated significant research in the past decade, thus it serves as a timely metric. Piezoceramic, along with other hysteretic materials such as shape memory alloys and ferromagnetic devices are candidates being chosen for cutting edge positioning applications in semiconductor production, computer disk drives, biomedical instruments and others. These materials are valued for their ability to directly convert electrical to mechanical forces at nanometer resolution without transmissions, which might introduce assembly or operating tolerances greater than the desired nanometer precision. D. A. Hall [6], has recently presented an excellent overview of the state of the understanding for piezoceramic actuators, and we recommend it as a primer.

The design details for commercial precision positioning controls using hysteretic actuators are not shared, but what is known is that they use calibrated models of the actuator to compensate for their linear Proportional + Integral (PI) controls. Research in the past two decades has focused almost exclusively on improving these calibration models and continued use of standard linear PI [7], loop shaping [8] and/or H-Infinity ($H\infty$) [9], control techniques. Some researchers modeling techniques have been straightforward linear frequency or time domain empirical models [7], while most have attempted to model the nonlinear hysteresis [7], [9], [10], [11].

Hysteresis compensation has generally followed three alternatives, all classical, and usually critically dependent on a precise model of hysteresis. One alternative is feed forward compensation, a similar alternative is a classic "prefilter" configuration, and a third popular alternative is the application of an inverse of the hysteresis nonlinearity in series with the process. In almost all instances, however, a model of the hysteresis has been integral to the success of the result, and much success controlling hysteresis has been a consequence of better hysteretic models, so that this research emphasis continues today.

Unfortunately, hysteresis compensation methods based on the prevailing phenomenological models are complicated to design and apply. The model development initially requires precise measurement of first order reversal curves for the specific hysteretic process being modeled, this may require tens of thousands or millions of data, dependent on the resolution one desires. These curves are also measured at some representative temperature, humidity and input power, the effects of which do not vary linearly, further complicating the math in real time. Each sample period calculation is also one of successive approximation of integral functions over large data sets, requiring significant computational resource to render in real time. All these complications taken together compound each other so that model based compensation, although achieving high resolution and precision, requires long compute cycles to resolve and must be calibrated and tuned regularly to account for nonlinear drift and offsets. In short, model based hysteresis compensation is very capable in a lab with expert practitioners, but it is not practical for casual

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use or high throughput.

There exist alternative models of hysteresis [12], [13] based on analytic functions, and though phenomenological models are more representative of measured data, analytic models are more mathematically tractable, so that when combined with adaptive control algorithms the results can be respectable [13], [14]. The issues surrounding these competing models then relate to the observations regarding hysteresis, the rate independent memory, multi valued transformation, device specific response, internal energy dissipation, etc.

This paper is organized as follows: The hysteresis compensation problem is reformulated and the simulation of the nonlinear hysteretic actuator model is described in section II. The treatment of hysteresis as disturbance and the description of the Active Disturbance Rejection Control (ADRC) is explained in section III. The demonstration criteria, performance constraints, design and tuning description, and graphical results are presented in section IV, along with comments on these results. Finally, a general conclusion and recommendation for further research is presented in section V.

II. A Reformulation Of Hysteresis Compensation And Demonstration Setup

The prevailing technique for hysteresis compensation has been model inversion, which achieves resolution and precision, but must be calibrated for each application and can be computationally intense to operate in real time. This research proposes an entirely different hysteresis compensation thesis: treat the nonlinear hysteresis as a common disturbance to the desired linear response, and actively reject that disturbance to present a compensated system to the controller which responds linearly, indeed as a simple double integrator. An extension of the classic disturbance observer is implemented wherein the effect of the hysteresis nonlinearity is treated as disturbance and canceled. The hysteresis will be compensated such that the resulting equivalent double integrator presented to the position controller may then be easily managed by a Proportional + Derivative (PD) control.

A. Piezoceramic

Qualitatively, a piezoceramic develops significant power for the mass, so inertia matching is a challenge from the start. The positive contribution from the direct energy to displacement conversion is balanced by an absence of viscous damping in a transmission, moving high frequency complex eigenvalues in the motion equations even closer to and farther along the imaginary axis. Exacerbating this is the elastic properties of the material, contributing harmonics of its natural resonant/anti-resonant pairs of eigenvalues, which can place the primary harmonic very near the force transmission eigenvalues. These complications are additional to the fundamental challenge of compensating hysteresis and yet are shared with other hysteretic materials, so their inclusion in this application example is purposeful.

An assumption is made in this research that the time scales for other nonlinearities of piezoceramic, such as thermal drift and voltage creep, are orders of magnitude slower than that of hysteresis and are herein modeled and controlled as slow additive and multiplicative variations in the linear gain parameters in the motion equations. The choice is made, in the spirit of singular perturbation theory but as assumption, to address hysteresis decoupled from the slower nonlinearities.

B. Hysteresis nonlinearity

Hysteretic response may be at times 20% or more nonlinear. The issues caused by this nonlinearity are exacerbated in many applications where the normal operating environment requires transiting the hysteretic region constantly and rapidly. A consequence of this hysteresis is the limited bandwidth of operation, achieving desired performance only at reduced operating speeds.

The understanding of hysteresis has been extensively enhanced recently [15], [16], [17], and the simulation development contained herein is based on the work of Brokate & Sprekels [15] and Krejci [17].

C. Semilinear and Quasilinear

Many researchers have evaluated strain displacement of the piezo device without external load. A more valid example applies a force to an external load, many times of greater mass than the actuator, which introduces 2 poles, usually very near the axis. Additionally, even those comparisons with an external load utilize a semilinear hysteresis model relying on the "equivalent" hysteresis damping coefficient, b_{eq}, in order to account for the energy dissipation. This model of dissipation is rate dependent, which is erroneous for hysteretic energy dissipation.

$$m\ddot{x}(t) + b_{eq}\dot{x}(t) + kx(t) = F(t) + P[x](t) \quad (1)$$

where the displacement $x(t)$ is a function of linear force $F(t)$, dissipative parameter b_{eq}, elastic constant k, and hysteresis operator $P[x](t)$.

A more recent alternative inverse hysteresis model utilizing the identity operator $I[x](t)$ and attributed to Della Torre [18] is quasilinear,

$$\begin{aligned} &m\ddot{x}(t) + kv(t) = F(t), \\ &x(t) = (I+P)[v](t), \\ &v(t) = (I+P)^{-1}[x](t) \end{aligned} \quad (2)$$

and thus appropriately accounts for the rate independent nature of hysteretic energy dissipation. In this model the displacement Force, F exerted on the load mass m is reduced by the actuator internal stored and dissipated force, here both rate independent and proportional to the elastic constant k.

A nonlinear Simulink model for the hysteretic actuator and load was used to demonstrate the controllers' effectiveness. The load model in Figure 1 utilized a quasilinear mechanical configuration with a Preisach operator for inverse hysteresis. These have been demonstrated to well represent the observed behavior of a hysteretic device, including the rate independent hysteretic energy dissipation [17]. Many hysteresis operators are based on area integrals using weighted basis functions, the Preisach operator uses a "relay" basis $R_{s-r,s+r}$. A Preisach operator then transforms a continuous piecewise monotonic input function $u(t)$ into another continuous piecewise monotonic output $w(t)$ as:

$$P[u](t) = w(t) = \int_0^{\infty}\int_{-\infty}^{\infty} \mu(r,s) R_{s-r,s+r}[u](t) ds dr \qquad (3)$$

where $\mu(r,s)$ is a nonnegative weighting function based on measurement for each specific device and assumed to vanish for large values of r and s. This piezoceramic actuator simulation is based on such actual measured weighting data and it is important to remark here on this constraint. The physical response of hysteretic processes is unique to each, and differ in symmetry, saturation, and swept area, but are generally similar in their anhysteretic small signal response and large signal saturation. Various hysteresis operators have strengths and weaknesses in their capabilities to accurately represent these differences, and this is critical when one is dependent on that model for control. However, the ability to represent the rate independent memory of hysteresis in the actuator model is necessary to best demonstrate the strength of any controller, thus the choice of the quasilinear model.

The proposed controller does not depend on any model of hysteresis, its stability and ability to reject disturbance depend on the Lipschitz continuity and boundedness of the disturbance [19], so the precision of the simulation model is not so critical, it should be reasonable for our purpose. It is thus beyond the scope of this paper to review the development of these hysteresis equations and operators, so the interested reader is encouraged to consult the references to confirm the choice.

The piezoceramic actuator was modeled as a simple tube placed with vertical centerline, as it alleviates the complexity of the cross coupling terms in the elastic equations. Different actuators, such as rods or cantilevered beams, would modify this model, recalling the strong coupling of the axes, and the hysteresis in each. This fact is not lost, and intuitively reinforces the treatment of hysteresis as an unknown disturbance to be measured and compensated in multiple axes rather than necessitating even more complex model based control calculations in real time. The demonstration of multi axis control is anticipated in future research.

The general elastic harmonic equation model in Fig. 1 is placed in series with the quasilinear load model in the simulation. The elastic properties of piezoceramic have been previously reported, they are high Q factor devices with a pronounced primary resonant peak, here a function of the quasilinear model component. The linear elastic model component includes the low frequency gain and second order pole and zero pairs. The output force of the tube, F, is a function of the controller input, u, and the physical attributes of the hysteretic material found in data sheets which determine the linear low frequency gain K and the linear coefficients a_0,a_1,a_2, and b_0,b_1,b_2. The reader is directed to Goforth [19], for specific coefficient equations for piezoceramic actuator as well as load mass m and elastic constant k used in this simulation example.

Any slow nonlinearity in the hysteretic material is modeled as general additive and multiplicative offsets respectively, assuming the time scales are long for their effects versus the linear elastic harmonics and hysteresis. The full motion equation can now be reformulated as:

$$\ddot{x}(t) = f(.) + bu(t) \qquad (4)$$

where $b=K/m$ and the various linear elastic terms and nonlinear contributions from hysteresis, creep, drift, etc. are combined as part of the generalized nonlinear function *f(.), simply denoted as f.*

This reformulation of the motion control problem in (4) leads us to a new solution shown in the next section. The key idea is the recognition that the control design does not have to necessarily depend on the exact mathematical expression of f, so long as its value can be estimated in real time from the input and output data, i.e. x and u. In fact, f can be treated as a general disturbance to the ideal, double integral, motion plant, as shown in Fig 1b. If a good estimate of f can be obtained from an observer, say the Linear Extended State Observer (LESO), then, by canceling f using its estimate, the control system is reduced to a simply PD design for a nominal double integral plant, shown within the dotted line in Fig. 2. Such design strategy of actively rejecting the unknown based on its real time estimation is known as the Active Disturbance Rejection Control (ADRC), as described below.. The author's thesis is that this strategy is particularly well suited for many difficult hysteretic applications, not just this application.

III. An Active Disturbance Rejection Control for Hysteresis Compensation

ADRC is a rather novel design methodology in its own right, and certainly novel for the application to a hysteretic process. It was originally proposed by by Han[20], [21] and extended by Gao [22]-[24] for the ease of implementation, in the form of Linear Active Disturbance Rejection Control (LADRC). The main idea is quite easy to understand: If f can be estimated, as $\hat{f}$, in real time, then a control law of

$$u = \frac{u_0 - \hat{f}}{b} \qquad (5)$$

reduces the plant in (4) to a double integral one:

$$\ddot{x} \approx u_0 \qquad (6)$$

which can be easily controlled using a PD controller.

Therefore the question comes down to whether or not f can be realistically estimated in real time.

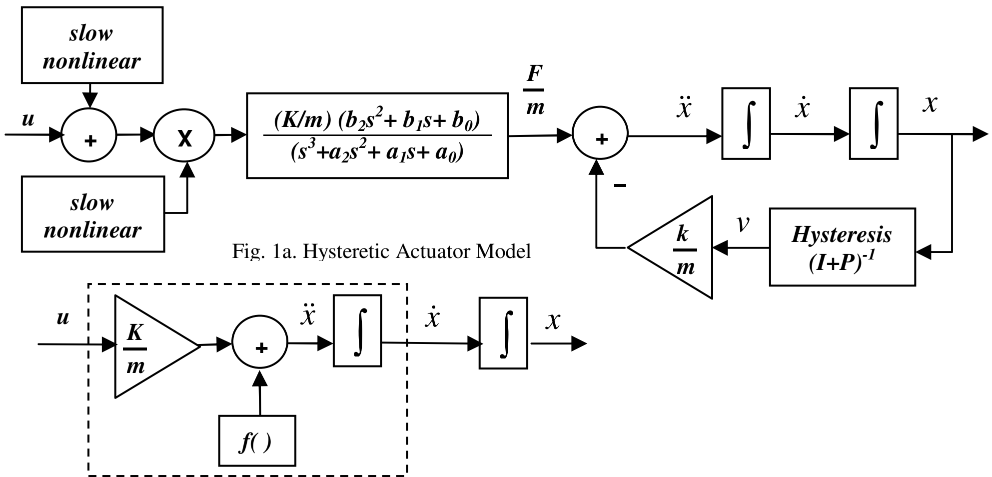

Fig. 1a. Hysteretic Actuator Model

Fig. 1b. Nonlinear Disturbance Model

Fig. 1. Hysteretic Actuator Model

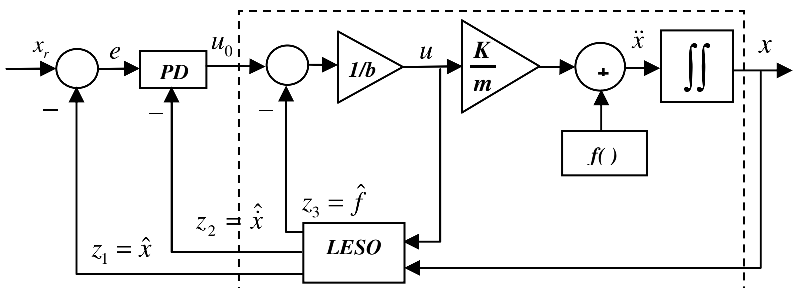

Fig. 2. Active Disturbance Rejection Control + PD position control

For the purpose of estimating f, we define an augmented state space representation of (4), with $x_1 = x,\ x_2 = \dot{x}, and\ x_3 = f$,

$$\begin{aligned} \dot{x} &= Ax + Bu + Eh \\ y &= Cx \end{aligned} \qquad (7)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (8)$$

The idea is that if f can be treated as a state variable, then perhaps it can be estimated in real time using a state observer of the form:

$$\begin{aligned} \dot{z} &= Az + Bu + L(y - \hat{y}) \\ \hat{y} &= Cz \end{aligned} \qquad (9)$$

with observer gain vector

$$L = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \end{bmatrix}^T \qquad (10)$$

to be determined.

Originally, Han proposed a generic nonlinear observer [19,20] which Gao simplified and parameterized to facilitate practical implementation. The result is is the Linear ESO (LESO) of the form of (9) whereby the observer gains are parameterized by placing all eigenvalues at ω_o, the observer bandwidth. The observer gains are then obtained from its characteristic equation

$$\begin{aligned} &\lambda_o(s) = s^3 + \beta_1 s^2 + \beta_2 s + \beta_3 = (s + \omega_o)^3 \\ &\Rightarrow \beta_1 = 3\omega_o,\ \beta_2 = 3\omega_o^2,\ \beta_3 = \omega_o^3 \end{aligned} \qquad (11)$$

Note that the observer design and tuning, in practice, is a trade-off between performance (speed) and noise sensitivity. The parameterization of the observer gains in (11) reduces the complexity in the task of tuning an observer to the act of adjusting one tuning parameter: ω_o. The resulting values of $z_1 = \hat{x},\ z_2 = \hat{\dot{x}},\ z_3 = \hat{f}$ and the control law:

$$u = \frac{u_0 - z_3}{b} \qquad (12)$$

will reduce the plant to:

$$\ddot{x} = (f - z_3) + u_0 \approx u_0 \qquad (13)$$

so that we have achieved our design goal and may choose among many controls appropriate for our double integrator equivalent plant.

A double integrator system is easily controlled by a Proportional + Derivative (PD) controller:

$$u_0 = K_p(r - x) - K_d \dot{x} \tag{14}$$

where one may simplify the choice of the gains K_p and K_d by placing both poles for the closed loop equivalent system at the same critically damped location:

$$G_{cl}(s) = \frac{\omega_c^2}{s^2 + 2\omega_c s + \omega_c^2} = \frac{K_p}{s^2 + K_d s + K_p} \tag{15}$$

$$\Rightarrow K_d = 2\omega_c,\ K_p = \omega_c^2$$

so that one may only concern oneself with tuning the controller bandwidth, ω_c. Note that x and $\dot{x}$ can be replaced by their estimates, z_1 and z_2, respectively, in the control law of (14). This implementation proves to be practical and effective in a wide range of applications [25]-[30]. Such an approach is referred to as the Linear ADRC (LADRC) implementation because of the use of the linear gains in ADRC.

I. THE LADRC IMPLEMENTATION AND HYSTERESIS COMPENSATION PERFORMANCE

We will apply the LADRC to the hysteretic actuator precision positioning simulation to demonstrate its capability to compensate for hysteresis. A description of the simulated dynamics of the actuator is followed by a description of the LADRC design and tuning procedure.

The simulation is normalized to stay within a range of reference $x_r = \pm 1$ *ref unit*. Fig. 3 is a plot of the test reference signal used for this demonstration, not including any of the disturbances, offsets, noise, etc.

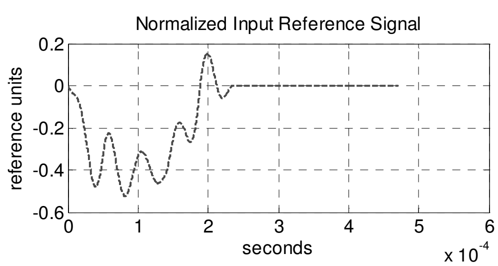

Fig. 3. Reference Signal

It must be noted the frequency of the reference signal at 20kHz compared to the 21kHz primary harmonic of the device which was simulated. This model also contains switches to add random electrical noise and step mechanical disturbances to both input and output signals of the actuator, plus nonlinear saturation of amplifier gains to test the controller response. The simulated random electrical noise impressed on the reference input and the feedback sensor is set at $1e^{-6}$ *ref units*. The control power amplifier is simulated to saturate at ±*1 control unit* and the simulated mechanical disturbance corresponded to ±*0.5 control unit* to compensate, or 50% power. External disturbances and noise are introduced during the transient periods and also during the quiescent input period of the reference signal.

A. Design and Tuning the LADRC

The one design parameter for the LADRC is the value of low frequency gain of the plant, in this case the actuator gain divided by load mass: $b=K/m$.

The complete implementation requires tuning of the observer and controller frequencies ω_o and ω_c. One may optimize tuning by definition of some cost function, or one may quickly arrive at a usable solution with the following heuristic steps.

First heuristic rule: choose a value for the observer sampling frequency which will assure accurate estimation of the nonlinear "disturbance". This is the most critical factor in the success of this strategy. Our heuristic is to sample at minimum one decade greater than the desired response speed of the actuator, and preferably 2 or more decades greater. The caveat should be obvious that noise becomes an upper limit. The second heuristic rule is to set the controller frequency $\omega_c = \omega_o/3$.

The calculated gain using data sheet values $b = 1.78e^{10}$ for the simulated actuator, the value which is programmed into the controller. The primary resonant frequency of the simulated system is calculated to be $4e^5/3$ *rad/sec* so that our heuristic tuning would indicate $\omega_o=4e^7/3$ *rad/sec and* $\omega_c=4e^7/9$ *rad/sec.*

B. Performance of the LADRC

Fig. 4a illustrates the internal inverse hysteresis response across the simulated actuator and Fig. 4b the closed loop system response using only LADRC(PD) control, the compensation for the hysteresis is almost linear.

Fig. 5 reveals the performance of the LADRC demonstration. The control signal illustrates the simulation offset disturbances at 20, 40 and 250 μsec (the impulses at 0, 50, 140 and 190 μsec are consequent of single computation period gaps in the reference signal simulation, they are unintended and yet well controlled by the LADRC). The simulated system experiences zero mean random noise injection of $\pm 1e^{-6}$ *ref units* at the reference input for ~25 μsec beginning at ~300 μsec, $\pm 1e^{-6}$ *ref units* at the output feedback for ~25 μsec beginning at ~350 μsec, and both sources simultaneously for ~25 μsec beginning at ~400 μsec, each random source has a different kernel. This noise level is consistent with radiated noise entering via the feedback measurement, and thermal noise and/or electrical noise in the reference input. The noise error $\sim 2e^{-5}/.7 = 0.003\%$, which is discernable because the observer frequency is higher and more measurement noise is injected into the system. The max error for the system is now $\sim 8e^{-4}/0.7 = 0.11\%$, for only a sample period and during the input discontinuities which have been previously mentioned. Most encouraging, both the nominal error at $\sim 3e^{-4}/0.7 = .043\%$ and disturbance error $<2e^{-4}/0.7 = .029\%$ are better than many model based controls. The control signal is less than one unit except during the discontinuities when still <*4 control units*. Additionally, due to the low computation bandwidth required, this control is capable at much higher operating frequencies.

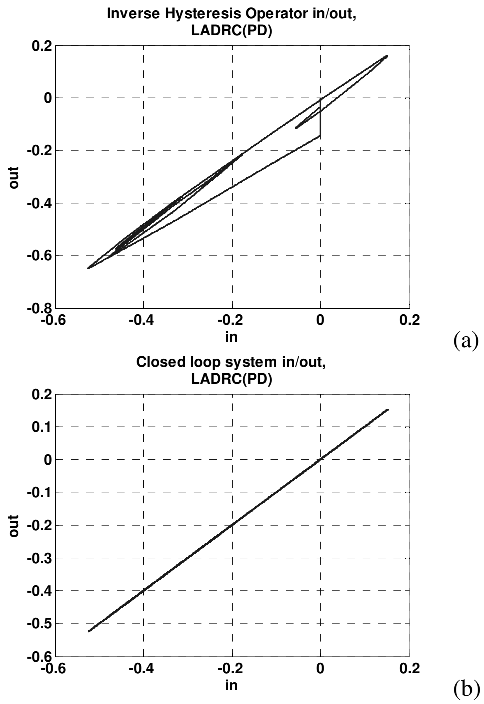

Fig. 4. Hysteresis Compensation Result

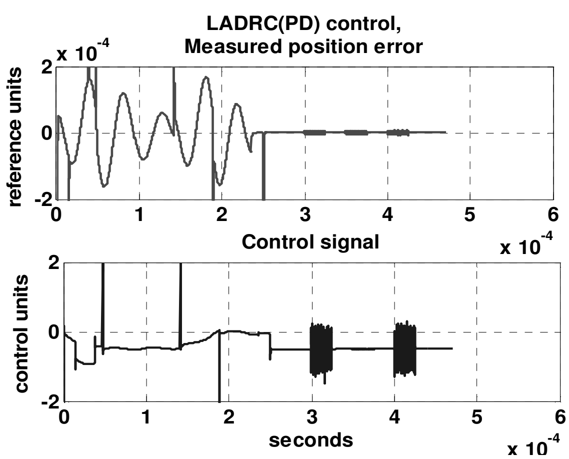

Fig. 5. LADRC(PD), position error and control effort

II. Conclusion

A novel method for hysteresis compensaiton is proposed whereby hysteresis is treated as a disturbance to the desired linear behavior and rejected. A hysteresis dominated motion control problem at nano scale, which has immediate and significant research interest, is chosen as an example to demonstrate the advantages of this new method. The positioning application performed by the piezoceramic actuator is one where the strengths of the disturbance rejection paradigm can be well appreciated. It is common for bioscience researchers, semiconductor equipment operators, medical lab technicians, etc. to spend hours acquiring a handful of sample measurements, constantly calibrating and compensating for the response limitations of model based controls. The hysteresis compensation problem is then generalized to stimulate the reader to consider the potential of the proposed solution being applied to other, perhaps dissimilar hysteretic processes.

The Linear Active Disturbance Rejection Control demonstration illustrates the superior ease of implementation of this disturbance rejection strategy in compensating for hysteresis, versus the model based paradigm, while achieving superior results in accuracy and operating bandwidth near to resonance. Additionally, this control requires minimal computational resource compared to model based hysteresis controllers, thus it may be implemented at less cost and/or higher response speed. The LADRC is designed and applied using classical linear data sheet parameters and tuning techniques. The nominal error to command of 0.04% and errors in the presence of disturbance, in this case hysteresis, of 0.03% are exceptional. The achievement of this performance without resorting to the complexity of mathematic modeling is a significant advance.

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