Fuzzy Estimation of DC Motor Winding Currents

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Abstract

Fuzzy systems have been used extensively and successfully in control systems over the past few decades, but have been applied much less often to filtering problems. This is somewhat surprising in view of the close relationship between control and estimation. This paper discusses and demonstrates the application of fuzzy filtering to motor winding current estimation in permanent magnet synchronous motors. It is shown via simulation results that fuzzy techniques provide an attractive alternative to current estimation using analytic techniques. It is further shown how the fuzzy membership functions can be optimized using gradient descent.

1 Introduction

The electrical windings of permanent magnet synchronous motors are spaced on the stator (the fixed part of the motor) at regular angles. When excited with current, the windings produce magnetic fluxes that add vectorially to produce the stator flux. The controlling variables are the proportions of currents in the motor windings, which determine flux magnitude and orientation. Rotating rotor magnets produce the rotor flux and interact with the stator flux to produce torque. When the stator and rotor fluxes are aligned, the magnetic fields are in equilibrium at the minimum energy position and no torque is produced. When the stator and rotor fluxes are not aligned, the rotor magnets are pulled toward the stator electromagnets. This torque is maximum when the rotor flux is 90° behind the stator flux in the direction of motion. At this point the flux vectors are said to be *field-oriented* for maximum torque at a given current. This is also the most efficient operating region of the motor, because in this mode the power input to the mechanical side of the motor is maximized. For continuous rotation at the highest torque and efficiency, the stator flux is rotated in the desired direction of motion, keeping 90° ahead of the rotor flux. The stator flux is produced by controlling the current in the stator windings.

In order to implement an effective closed-loop current controller we need an accurate estimate of the current [2]. Current estimation is thus an important problem. It is also a challenging problem because the measured winding currents are strongly affected by electrical noise in the motor electronics.

The motor's winding currents are generally shaped like a sinusoid. Knowing this, we can formulate common-sense fuzzy membership functions for use in a predictor-corrector type of estimator. These initial membership functions are constructed on the basis of experience, and trial and error. The fuzzy system is a recursive nonlinear estimator. Its inputs are comprised of past estimates, and present and past measurements. From this starting point we gather experimental data from a motor and fine-tune the fuzzy membership functions using gradient-descent. The fuzzy estimator is applied to actual motor winding currents. The results presented in this paper establish fuzzy estimation as a viable alternative for stator winding current estimation.

2 Fuzzy Estimation

We begin with a standard discrete, time-invariant system given by

$$x_{k+1} = f(x_k) + v_k \quad (1)$$
$$z_k = h(x_k) + w_k \quad (2)$$

where k is the time index, x_k is the state vector, z_k is the measurement, and v_k and w_k are noise processes. The problem of finding an estimate $\hat{x}_k$ for the state vector based on past and present measurements is known as the *a posteriori* filtering problem. One commonly used estimator architecture is the recursive predictor/corrector, given by

$$\hat{x}_k = \hat{f}(\hat{x}_{k-1}) + g(z_k, \hat{x}_{k-1}) \quad (3)$$

where $\hat{f}(\cdot)$ is an estimate of $f(\cdot)$, and $g(\cdot)$ is the correction function. The process model $f(\cdot)$ is often known, or it can be found using system identification methods. If $\hat{f}(\cdot)$ is available, only the correction mapping $g(\cdot)$ needs to be determined. Various analytic methods have been used for obtaining the correction mapping [5, 7]. As an alternative to analytic methods, the correction mapping could be implemented as a fuzzy function [6].

2.1 Current Estimation

Consider the problem of tracking a noisy discrete-time signal $x(n)$. The fuzzy estimator structure that we use to obtain an estimate of the signal is given by

$$\hat{x}_k^- = \hat{x}_{k-1}^+ + T\hat{v}_{k-1} \quad (4)$$
$$\hat{x}_k^+ = g(z_k, \hat{x}_k^-) \quad (5)$$

$\hat{x}_k^-$ denotes the estimate of x at time k *before* the measurement at time k is processed. $\hat{x}_k^+$ denotes the estimate of x at time k *after* the measurement at time k is processed. T is the update period of the estimate, z_k is the noisy measurement of the winding current, and $\hat{v}$ is the estimate of current rate. (The determination of the rate estimate is discussed in Section 2.2.) The fuzzy correction mapping $g(\cdot)$ has two inputs:

$$(\text{input } 1)_k = z_k - \hat{x}_k^- \quad (6)$$
$$(\text{input } 2)_k = (\text{input } 1)_k - (\text{input } 1)_{k-1}. \quad (7)$$

The output of the correction mapping is a fuzzy variable which is determined by correlation-product inference.

The initial rule base and triangular membership functions were constructed on the imprecise basis of experience, and trial and error. An appropriate initial knowledge base is critical, because without an initial knowledge we cannot proceed any further with any optimization schemes. In spite of its importance, the generation of initial knowledge remains as a difficult and ill-defined task in the construction of fuzzy logic systems.

In general, we denote the centroid and half-width of the i-th fuzzy membership function of the j-th input by c_{ij} and b_{ij}. So the degree of membership of a crisp input x in the i-th category of the j-th input is given by

$$f_{ij}(x) = \begin{cases} 1- \mid x - c_{ij} \mid /b_{ij} & \mid x - c_{ij} \mid \leq b_{ij}/2 \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

The fuzzy output is mapped into a crisp numerical value using centroid defuzzification [4].

$$g(z_k, \hat{x}_k^-) = \frac{\sum_{j=1}^{n} m(y_j) y_j J_j}{\sum_{j=1}^{n} m(y_j) J_j} \quad (9)$$

where y_j and J_j are the centroid and area of the j-th output fuzzy membership function and n is the number of fuzzy output sets. (Note that in the case of symmetric triangular membership functions, J_j is equal to the half-width of the j-th output fuzzy membership function.) The fuzzy output function $m(y)$ is computed as

$$m(y) = \text{fuzzy output function} = \sum_{i,k} m_{ik}(y) \quad (10)$$

$$m_{ik}(y) = \text{fuzzy output function when} \quad (11)$$
$$(\text{input } 1 \in \text{class } i) \text{ and } (\text{input } 2 \in \text{class } k)$$

2.2 Current Rate Estimation

One of the inputs to the fuzzy estimator discussed above is the current rate estimate $\hat{v}$. This estimate must be computed from the current estimates using numerical differentiation, which is in itself a challenging task. We will somewhat arbitrarily assume that we have the present and past three estimates available. With this in mind, we use the method of undetermined coefficients [1] to obtain the following expression for the rate.

$$\begin{aligned} v(t) &= [-\frac{1}{3}x(t-3\tau) + \frac{3}{2}x(t-2\tau) - 3x(t-\tau) \\ &\quad + \frac{11}{6}x(t)]/\tau - \frac{193\tau^3}{72}x^{(4)}(\zeta) \end{aligned} \quad (12)$$

where τ is a time step (some multiple of T in Section 2.1) to be determined later, and ζ is an unknown constant in $[t-3\tau, t]$. It is our objective in the remainder of this section to determine an appropriate time step τ. If we estimate v in accordance with the above equation, then we can perform the error analysis associated with the Taylor Series expansions of $\hat{x}(t-k\tau)$ to obtain the following expression for the rate estimation error:

$$\begin{aligned} \tilde{v}(t) &= \frac{-193\tau^3}{72}x^{(4)}(\zeta) + [-\frac{1}{3}\tilde{x}(t-3\tau) \\ &\quad + \frac{3}{2}\tilde{x}(t-2\tau) - 3\tilde{x}(t-\tau) + \frac{11}{6}\tilde{x}(t)]/\tau. \end{aligned} \quad (13)$$

Now if we treat the time functions in the above equation as random processes and make the simplifying assumption that $x^{(4)}(t)$ and $\tilde{x}(t)$ are independent, we obtain the following expression for the variance of the rate estimation error:

$$E(\tilde{v}^2) = \left(\frac{193\tau^3}{72}\right)^2 E([x^{(4)}]^2) + \frac{530}{36\tau^2}E(\tilde{x}^2). \quad (14)$$

Based on our knowledge of the current waveforms, we will assume that we have a one-sigma current estimation error that corresponds to about 0.01 volts. (Note that the current is measured by an analog-to-digital converter [ADC] on the motor drive, so the acquired voltage is directly proportional to the motor winding current.) Again, based on our knowledge of the current waveforms, we will assume that the standard deviation of the fourth derivative of the current is 2.9 μV/(ms)^4. Our ADC operates at a rate of 200 μs, so τ must be a multiple of 200 μs. We can then minimize the rate estimation variance with respect to τ. Doing so, we obtain the surprising result that the variance of the rate estimation error is minimized for $\tau \approx 35T$. In fact, the rate estimation error is strongly dependent on τ. For instance, it can be shown that the rate error is approximately 1000 times greater for $\tau = T$ than for $\tau = 35T$. So we will use $\tau = 35T$ (where $T = 200$ μs is the ADC period) in (4) to estimate rate. This finding is critical to the success of the fuzzy estimator.

3 Optimization

If the fuzzy membership functions are triangular, gradient descent can be used to optimize the centroids and the widths of the input membership functions, and the centroids of the output membership functions [3, 6]. Consider an error function given by

$$\begin{aligned} E &= \frac{1}{2N}\sum_{q=1}^{N} E_q^2 \quad (15) \\ E_q &\equiv \hat{x}_q - x_q \end{aligned}$$

where N is the number of training samples. We can optimize E by using the partial derivatives of E with respect to: (a) the centroids of the input fuzzy membership functions; (b) the half-widths of the input fuzzy membership functions; and (c) the centroids of the output fuzzy membership functions.

Input Centroids

Using the relationships of (8) and following, we obtain

$$\frac{\partial E}{\partial c_{ij}} = \frac{1}{N}\sum_{q=1}^{N} E_q \frac{\partial \hat{x}_q}{\partial c_{ij}} \quad (16)$$

$$\frac{\partial \hat{x}_q}{\partial c_{ij}} = \sum_{p=1}^{n} \frac{\partial \hat{x}_q}{\partial m_p}\frac{\partial m_p}{\partial c_{ij}} \qquad [m_p \equiv m(y_p)] \quad (17)$$

$$\frac{\partial \hat{x}_q}{\partial m_j} = \frac{J_j(y_j - \hat{x}_q)}{\sum_{i=1}^{n} m_i J_i} \quad (18)$$

$$\frac{\partial m_p}{\partial c_{ij}} = \sum_{k,l} r_{klp}\frac{\partial w_{kl}}{\partial c_{ij}} \quad (19)$$

where $r_{klp} = 1$ if [(input 1) $\in$ class k and (input 2) $\in$ class l] $\Longrightarrow$ (output $\in$ class p), and 0 otherwise.

$\partial w_{kl}/\partial c_{ij}$ is given as follows:

$$\frac{\partial w_{kl}}{\partial c_{ij}} = \begin{cases} \partial f_{k1}/\partial c_{ij} & f_{k1}(\text{input } 1) \leq f_{l2}(\text{input } 2) \\ \partial f_{l2}/\partial c_{ij} & \text{otherwise.} \end{cases}$$

The partials of the membership grades $f(\cdot)$ with respect to the input centroids are

$$\frac{\partial f_{k1}(\cdot)}{\partial c_{i2}} = \frac{\partial f_{l2}(\cdot)}{\partial c_{i1}} = 0 \quad (20)$$

$$\frac{\partial f_{k1}(\cdot)}{\partial c_{i1}} = 2\delta_{ik} \text{ sign } [(\cdot) - c_{i1}]/b_{i1} \quad (21)$$

$$\frac{\partial f_{l2}(\cdot)}{\partial c_{i2}} = 2\delta_{il} \text{ sign } [(\cdot) - c_{i2}]/b_{i2} \quad (22)$$

where δ_{ik} is the Kronecker delta function ($\delta_{ik} = 1$ for $i = k$, 0 otherwise).

Input Half-Widths

Again using (8) and following, it can be shown that

$$\frac{\partial E}{\partial b_{ij}} = \frac{1}{N}\sum_{q=1}^{N} E_q \frac{\partial \hat{x}_q}{\partial b_{ij}} \quad (23)$$

$$\frac{\partial \hat{x}_q}{\partial b_{ij}} = \sum_{p=1}^{n} \frac{\partial \hat{x}_q}{\partial m_p}\frac{\partial m_p}{\partial b_{ij}} \qquad [m_p \equiv m(y_p)] \quad (24)$$

$$\frac{\partial \hat{x}_q}{\partial m_j} = \frac{J_j(y_j - \hat{x}_q)}{\sum_{i=1}^{n} m_i J_i} \quad (25)$$

$$\frac{\partial m_p}{\partial b_{ij}} = \sum_{k,l} r_{klp}\frac{\partial w_{kl}}{\partial b_{ij}} \quad (26)$$

where r_{klp} is given is given following (19) and $\partial w_{kl}/\partial b_{ij}$ is given by

$$\frac{\partial w_{kl}}{\partial b_{ij}} = \begin{cases} \partial f_{k1}/\partial b_{ij} & \text{if } f_{k1}(\text{input } 1) \leq f_{l2}(\text{input } 2) \\ \partial f_{l2}/\partial b_{ij} & \text{otherwise.} \end{cases} \quad (27)$$

The partials of the membership grades with respect to the half-widths of the input fuzzy membership functions are given as

$$\frac{\partial f_{k1}(\cdot)}{\partial b_{i2}} = \frac{\partial f_{l2}(\cdot)}{\partial b_{i1}} = 0 \quad (28)$$

$$\frac{\partial f_{k1}(\cdot)}{\partial b_{i1}} = \delta_{ik}[1 - (\cdot)]/b_{i1} \quad (29)$$

$$\frac{\partial f_{l2}(\cdot))}{\partial b_{i2}} = \delta_{il}[1 - (\cdot)]/b_{i2} \quad (30)$$

Output Centroids

The partials of the objective function E with respect to the centroids of the output fuzzy membership functions are given as

$$\frac{\partial E}{\partial y_j} = \frac{1}{N}\sum_{q=1}^{N} E_q \frac{\partial \hat{x}_q}{\partial y_j} \quad (31)$$

$$\frac{\partial \hat{x}_q}{\partial y_j} = \frac{m_j J_j}{\sum_{i=1}^{n} m_i J_i}. \quad (32)$$

The gradient descent rule is then used to update the independent variables from one iteration to the next as follows:

$$c_{ij}(k+1) = c_{ij}(k) - \eta_c \frac{\partial E(k)}{\partial c_{ij}} \quad (33)$$

$$b_{ij}(k+1) = b_{ij}(k) - \eta_b \frac{\partial E(k)}{\partial b_{ij}} \quad (34)$$

$$y_j(k+1) = y_j(k) - \eta_y \frac{\partial E(k)}{\partial y_j} \quad (35)$$

where η_c, η_b and η_y are gradient descent step sizes.

4 Simulation Results

The gradient descent method described above was used to optimize the fuzzy membership functions. The training data consisted of real motor winding currents collected with a digital oscilloscope at a rate of one sample every 200 μs which were run through a simple symmetric noncausal moving average filter consisting of a total of 51 points. The gradient descent learning parameters η_c, η_b and η_y were all initialized to 1. The membership functions were constrained to be symmetric triangles, and the error function E in (15) was optimized with respect to 35 parameters – the centroids of each of the membership functions (21 total), and the width of each of the input membership functions (14 total).

Figure 1 shows 2500 samples of raw current. (The vertical axis is Volts because the current is acquired with an ADC, which measures the current with a proportional voltage.) Figure 2 shows the same current after being passed through a fuzzy filter that was optimized via gradient descent. The improved smoothness of the filtered current is evident from a comparison of the figures. This will result in an improvement of the current control scheme for the DC motor.

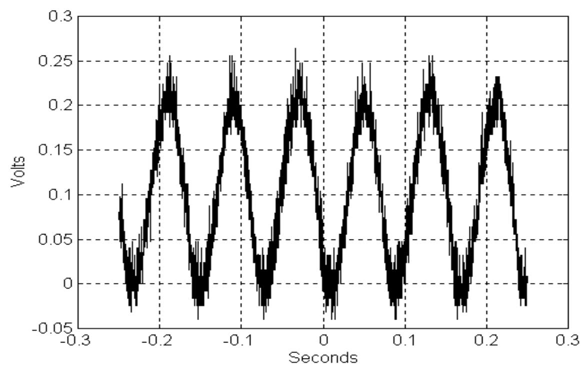

Figure 1: Unfiltered Currents

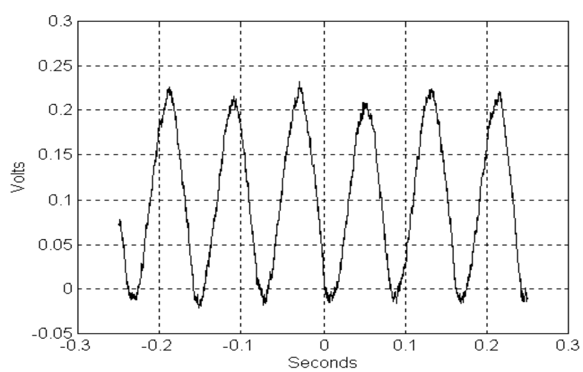

Figure 2: Filtered Currents

One potential drawback of the fuzzy estimator relative to analytic filters is the increased real-time computational effort. Most of the computational time of analytic filters consists of simple multiply and accumulates, which are straightforward to implement in a real-time digital signal processor or microcontroller. Fuzzy filtering, however, includes fuzzification, correlation-product inference, and defuzzification, and is therefore significantly more time-consuming.

5 Conclusion

A fuzzy estimator has been applied to the estimation of DC motor winding currents. This approach offers the benefits of fuzzy logic while providing performance on par with analytic methods. The fuzzy estimator also offers the possibility of training if a nominal current history is known *a priori*. Further work could focus on implementation issues in a real-time motor controller, and other optimization methods (e.g., genetic algorithms).

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