A Robust Two-Degree-of-Freedom Control Design Technique and its Practical Application

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***Abstract*— This paper introduces a new robust two-degree-of-freedom (2DOF) control design technique that extends the concepts of Active Disturbance Rejection Control (ADRC) and Proportional-Integral-Derivative (PID) control in new directions. The straightforward tuning of one or two parameters gives the technique its practicality. Together with ADRC, these techniques form a set of powerful design tools for direct use in industrial control applications. A method for controlling systems with finite zeros is also proposed. A simulation performed on an actual motion control platform provides verification and insight.**

***Keywords*— ADRC, Model Independent, Tuning, Robustness, Disturbance Rejection, State Observer, 2DOF, PID, I-PD**

I. INTRODUCTION

A. Background

Since its inception over eighty years ago [1], the proportional-integral-derivative (PID) control structure has remained the most commonly used single-input single-output (SISO) technique in industry, primarily because it is one of the simplest [2,3]. On the other hand, modern design structures often use linear-time-invariant (LTI) models to approximate physical systems that are time-varying and nonlinear in practice. The lack of efficient tuning techniques and the general need for the end-user to have an extensive background in control theory prevents model-based techniques from being found in industrial control applications. In this paper, the focus is turned from the development of complex control structures to that of a model-independent control framework and insightful tuning techniques.

Active Disturbance Rejection Control (ADRC) was originally proposed as a nonlinear solution to industrial control problems [4-6], but the large number of gains made tuning an art. The structure was simplified to its linear form and parameterized down to a few gains in [7]. It has been applied to many practical problems [8-11]. As a natural extension of the concepts behind ADRC and PID, this paper introduces a new robust two-degree-of-freedom (2DOF) control design technique. Parameterization is geared toward the ease of tuning in the industrial environment.

The paper is organized as follows. The remainder of Section I presents a brief overview of ADRC from [7], providing the foundation for discussion and comparison to the new material. Section II proposes a robust 2DOF control design technique and supplies an example of model-based parameterization. Section III analyzes the stability of the closed loop system and demonstrates robustness through a model-independent reformulation of the control structure. A method for controlling systems with finite zeros is also proposed. In Section IV, the 2DOF technique is verified in simulation. The results are then compared with ADRC. Finally, Section V offers concluding remarks.

B. Linear ADRC – A Second Order Application

For the sake of simplicity, consider the differential equation of a second order plant

$$\ddot{y} = w - a_1\dot{y} - a_0 y + b_0 u \tag{1}$$

where u and y are the input and output respectively. The external disturbance, w, along with all known and unknown dynamics, $-a_1\dot{y} - a_0 y + (b_0 - b)u$, are combined to form the generalized disturbance, $f(y, \dot{y}, w, u)$ or simply f. It represents the true dynamic behavior of the physical system. The plant is then rewritten as

$$\ddot{y} = f + bu\,. \tag{2}$$

Here, all of the parameters are unknown, with the exception of b_0, approximated by b from the initial acceleration of a step response. The idea behind ADRC is to estimate and cancel f in real time, reducing the plant to a double-integrator control problem. To do so, the plant is written with an additional state to track f

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + bu \\ \dot{x}_3 = \dot{f} \end{cases} \tag{3}$$

where $x_1 = y,\quad x_2 = \dot{y},\quad x_3 = f$. Based on (3), a state observer is constructed

$$\begin{aligned} \dot{z} &= Az + Bu + L(y - \hat{y}) \\ \hat{y} &= Cz \end{aligned} \tag{4}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix},\quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

where $z \to x$. The observer reduces to the following set of state equations, and is referred to as an extended state observer (ESO).

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$$\begin{cases} \dot{z}_1 = z_2 + L_1(y - z_1) \\ \dot{z}_2 = z_3 + L_2(y - z_1) + bu \\ \dot{z}_3 = L_3(y - z_1) \end{cases} \tag{5}$$

By setting $\lambda(s) = |sI - (A - LC)| = s^3 + L_1 s^2 + L_2 s + L_3$ equal to the desired error dynamics, $(s + \omega)^3$, the observer gains are solved as functions of a single tuning parameter, ω_o.

$$L_1 = 3\omega_o, \quad L_2 = 3\omega_o^2, \quad L_3 = \omega_o^3 \tag{6}$$

As $z_3 \to f$, it is used to actively cancel f by applying

$$u = (u_0 - z_3)/b. \tag{7}$$

This reduces the plant to $\ddot{y} = (f - z_3) + u_0 \approx u_0$, a unit gain double integrator, allowing a 2DOF PD controller to be used.

$$u_0 = k_p(r - z_1) - k_d z_2 \tag{8}$$

Given that the set point r is only present in the proportional term, an approximate closed loop transfer function is created without the addition of zeros from the controller.

$$\frac{y(s)}{r(s)} \approx \frac{k_p}{s^2 + k_d s + k_p} \tag{9}$$

By setting (9) equal to the desired transfer function, $\omega_c^2/(s + \omega_c)^2$, the controller gains are solved as functions of one tuning parameter, ω_c.

$$k_p = \omega_c^2, \quad k_d = 2\omega_c \tag{10}$$

Remarks

1. The entire structure has three parameters b_0, ω_o, and ω_c, which can be individually specified or tuned.
2. The ESO is further simplified by substituting (7) into (5), to remove an algebraic loop and decouple z_3, allowing ADRC to be represented in PID form.

$$u = (k_p(r - z_1) - k_d z_2 - L_3 \int (y - z_1))/b \tag{11}$$

3. To show how z_3 converges to f, it is clear from (2) that $f = \ddot{y} - b_0 u$. After solving (5), (7), and (8) for z_3 by superposition, the result is simply a filtered version of f.

$$z_3(s) = \left(s^2 y(s) - b_o u(s)\right) \frac{\omega_o^3}{(s + \omega_o)^3} \tag{12}$$

C. *Linear ADRC – An n^{th} Order Application*

An n^{th} order plant can be represented in the following canonical form

$$y^{(n)} = f + bu. \tag{13}$$

To apply ADRC to this type of plant, the extended state observer is given as

$$\begin{cases} \dot{z}_1 = z_2 + L_1(y - z_1) \\ \vdots \\ \dot{z}_{n-1} = z_n + L_{n-1}(y - z_1) \\ \dot{z}_n = z_{n+1} + L_n(y - z_1) + bu \\ \dot{z}_{n+1} = L_{n+1}(y - z_1) \end{cases} \tag{14}$$

where

$$\left\{ L_j = \binom{n+1}{j} \omega_o^j \right\} \quad j = 1, 2, \cdots n+1 \tag{15}$$

and $\binom{n}{j}$ denotes the binary coefficients of Pascal's Triangle [12]. The control law is given by

$$u = k_0(r - z_1) - k_1 z_2 - \cdots - k_{n-1} z_n - z_{n+1} \tag{16}$$

where

$$\left\{ k_j = \binom{n}{j} \frac{\omega_c^{n-j}}{b} \right\} \quad j = 0, 1, \cdots n-1. \tag{17}$$

II. A GENERALIZED 2DOF CONTROL STRUCTURE

PID is used in over ninety percent of all industrial control applications. Although it is simple to use, PID is limited in performance, particularly when dealing with dynamic uncertainties. The difficulty of tuning is an obstacle in implementation. Even the widely used pole placement technique is not straightforward because PID controllers operate by adding zero(s) to the loop. Based on ADRC, the PID structure is reformulated to make it more robust, easier to tune, and extendable to n^{th} order. For the sake of clarity, the ideas are conveyed using a few useful examples.

Consider the unity feedback control structure in Fig. 1, with a plant G(s) and a controller C(s).

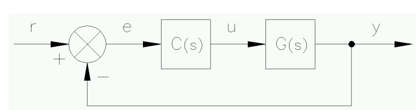

Figure 1. Unity Feedback Control Structure

The traditional PID controller is defined as a polynomial.

$$C(s) = k_i s^{-1} + k_p s^0 + k_d s^1 \tag{18}$$

A generalized PID controller is now proposed.

$$C(s) = k_{-m} s^{-m} + \cdots + k_{-1} s^{-1} + k_0 s^0 + k_1 s^1 + \cdots + k_n s^n \tag{19}$$

In this configuration, the zeros of C(s) are added to the zeros of the closed loop transfer function, making the tuning of C(s) a daunting task and prone to cancellation.

To deal with this fundamental design issue, the following generalized 2DOF control structure is proposed

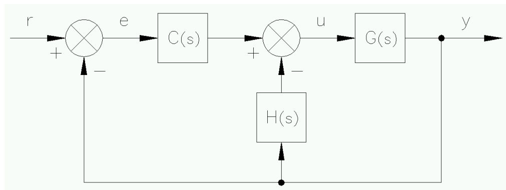

Figure 2. 2 DOF Control Configuration

where

$$C(s) = k_{j-1}s^{j-1}, \quad H(s) = k_j s^j + \cdots + k_{n-1}s^{n-1}. \quad (20)$$

and the integer j is less than or equal to the number of pure integrators in the plant. For a particular plant, this 2DOF control law represents a set of controllers that do not generate additional zeros in the closed loop transfer function. For example, Table I illustrates a number of controllers suitable for use with a double integrator plant.

$$G(s) = b_0 / s^2$$

TABLE I. 2DOF CONTROLLERS FOR A DOUBLE INTEGRATOR

j	C(s)	H(s)	Gains	y(s)/r(s)
2	$k_d s$	–	$k_d = \omega_c / b$	$\frac{\omega_c}{(s+\omega_c)}$
1	k_p	$k_d s$	$k_p = \omega_c^2 / b$ $k_d = 2\omega_c / b$	$\frac{\omega_c^2}{(s+\omega_c)^2}$
0	$k_i s^{-1}$	$k_p + k_d s$	$k_i = \omega_c^3 / b$ $k_p = 3\omega_c^2 / b$ $k_d = 3\omega_c / b$	$\frac{\omega_c^3}{(s+\omega_c)^3}$
-1	$k_{-2}s^{-2}$	$k_i s^{-1} + k_p + k_d s$	$k_{-2} = \omega_c^4 / b$ $k_i = 4\omega_c^3 / b$ $k_p = 6\omega_c^2 / b$ $k_d = 4\omega_c / b$	$\frac{\omega_c^4}{(s+\omega_c)^4}$

A closer look shows when *j=0*, a single integrator in C(s) ensures zero steady state error while improving disturbance rejection. When *j>0*, the controller is more prone to disturbances and noise. When *j<0*, disturbance rejection improves slightly, but at the expense of added phase lag. As a result, the 2DOF control law reduces to

$$C(s) = k_{-1}s^{-1}, \quad H(s) = k_0 s^0 + \cdots + k_{n-1}s^{n-1}. \quad (21)$$

Referred to as the generalized I-PD controller or simply GIPD, it can be viewed as a form of state feedback with integral control [13], where the states are defined specifically by the plant output and its (n) derivatives.

Example 1

Consider an LTI plant in the form

$$G(s) = \frac{b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0 s^0}. \quad (22)$$

The GIPD controller is applied, generating the following closed loop transfer function.

$$\frac{y(s)}{r(s)} = \frac{b_0 k_{-1}}{s^{n+1} + (b_0 k_{n-1} + a_{n-1})s^n + \cdots + (b_0 k_0 + a_0)s + b_0 k_{-1}} \quad (23)$$

By setting it equal to the desired transfer function, $\omega_c^{n+1} / (s+\omega_c)^{n+1}$, the controller gains are solved as functions of bandwidth and known plant parameters.

$$\left\{ k_j = \binom{n+1}{n-j} \frac{\omega_c^{n-j}}{b} - \frac{\bar{a}_j}{b} \right\} \; j = -1, 0, \cdots, n-1 \quad (24)$$

Here, $\bar{a}_j \; j \in [0, n-1]$ are estimates of the plant denominator coefficients, $a_j \; j \in [0, n-1]$. Table II gives examples of generalized I-PD controllers for first through third order plants.

TABLE II. EXAMPLES OF PARAMETERIZED 2DOF I-PD CONTROLLERS

G(s)	C(s)	H(s)	Gains
$\frac{b_0}{s + a_0}$	$k_i s^{-1}$	k_p	$k_i = \omega_c^2 / b$ $k_p = (2\omega_c - \bar{a}_0)/b$
$\frac{b_0}{s^2 + a_1 s + a_0}$	$k_i s^{-1}$	$k_p + k_d s$	$k_i = \omega_c^3 / b$ $k_p = (3\omega_c^2 - \bar{a}_0)/b$ $k_d = (3\omega_c - \bar{a}_1)/b$
$\frac{b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$	$k_i s^{-1}$	$k_p + k_d s + k_2 s^2$	$k_i = \omega_c^4 / b$ $k_p = (4\omega_c^3 - \bar{a}_0)/b$ $k_d = (6\omega_c^2 - \bar{a}_1)/b$ $k_2 = (4\omega_c - \bar{a}_2)/b$

Remarks

1. With an unknown plant, the $\bar{a}_j$ parameters can be tuned from zero to achieve the desired damping.
2. Tuning parameters directly affect either bandwidth or damping, making the process of tuning transparent.

III. STABILITY AND ROBUSTNESS

A. *Stability*

The stability of the controlled system is now analyzed. The closed loop response in (23) is parameterized with (24), producing a characteristic polynomial

$$\lambda(s) = s^{n+1} + d_n s^n + \cdots + d_0$$

where
$$d_{j+1} = \Delta b \binom{n+1}{n-j} \omega_c^{n-j} - \Delta b \bar{a}_j + a_j \qquad (25)$$

and $\Delta b = b_0 / b$ when b_0 is not precisely known.

A necessary condition for the stability of an LTI system is that all coefficients of $\lambda(s)$ are non-negative. Necessary and sufficient conditions are generated from the Routh-Hurwitz criterion [14, 15], resulting in a set of inequalities that place n-dimensional limits on plant coefficients, given ω_c and $\bar{a}_j$.

For second order plants, the condition $d_2 > d_0/d_1$ creates a curve, forming the boundary of a stability region to include all stable, as well as some unstable plant coefficients. Fig. 3 shows how the region expands proportional to the bandwidth of the plant when the controller bandwidth is increased.

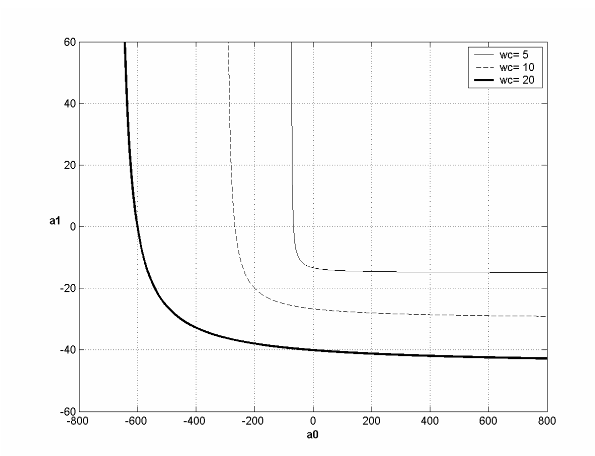

Figure 3. Stability Bound for Second Order Plants

For third order plants, two conditions, $d_3 > d_1/d_2$ and $d_2 > d_1/d_3 + d_0 d_3/d_1$, form a boundary surface. The first inequality is used to partition the surface generated by the second inequality, the more restrictive of the two conditions. Fig. 4 shows an example of such a surface.

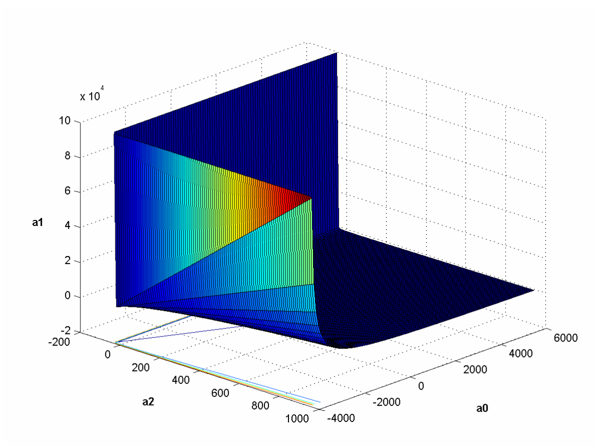

Figure 4. Stability Bound for Third Order Plants

As a convention, ω_c and Δb are positive in all cases. First through third order plants are tabulated in Table III, demonstrating a pattern of stability.

TABLE III. STABILY BOUNDS FOR GENERALIZED I-PD CONTROLLERS

n	Boundary	Type
1	$a_0 > -\Delta b(2\omega_c - \bar{a}_0)$	interval
2	$a_1 > \dfrac{\Delta b \omega_c^3}{\Delta b(3\omega_c^2 - \bar{a}_0) + a_0} - \Delta b(3\omega_c - \bar{a}_1)$	curve
3	$a_2 > \dfrac{\Delta b(4\omega_c^3 - \bar{a}_0) + a_0}{\Delta b(6\omega_c^2 - \bar{a}_1) + a_1} - \Delta b(4\omega_c - \bar{a}_2)$ $a_1 > \dfrac{\Delta b(4\omega_c^3 - \bar{a}_0) + a_0}{\Delta b(4\omega_c - \bar{a}_2) + a_2} + \dfrac{\Delta b \omega_c^4(\Delta b(4\omega_c - \bar{a}_2) + a_2)}{\Delta b(4\omega_c^3 - \bar{a}_0) + a_0} - \Delta b(6\omega_c^2 - \bar{a}_1)$	surface

<u>Remarks</u>

1. The first term in each coefficient of $\lambda(s)$ is dominant.
2. The structure produces stable results over a broad range without compromising steady state error, overshoot, or disturbance rejection properties.

B. *Robustness and Disturbance Rejection*

Robustness is a measure of how efficient the controller is in compensating for dynamic uncertainties inside the plant. On the other hand, disturbance rejection is a measure of how efficient the controller is in rejecting external disturbances. Although they are separate quantities, perhaps robustness and disturbance rejection are both achieved by overcoming f, given that internal uncertainties and external disturbances are both contained within f. The key concept behind ADRC is to estimate and cancel f. In this paper, the idea is to study how much a particular control structure can reject f and be parameterized without knowledge of f.

Any SISO plant, including those with finite zeros, time delay, and other nonlinearities can be represented as

$$y^{(n)} = f + bu . \qquad (26)$$

By combining all of the unknowns, model-independent parameterization and analysis can take place. With this in mind, the 2DOF control configuration is re-formulated by representing the plant in this way.

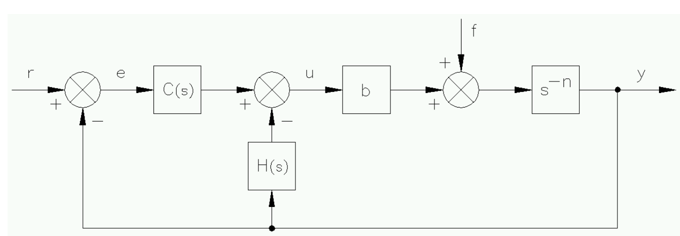

Figure 5. 2DOF Control Structure Reformulated

Given a system's order, the GIPD control law is applied. Using superposition, the closed loop response is derived.

$$\frac{y(s)}{r(s)} = \frac{bk_i}{s^{n+1} + bk_{n-1}s^n + \cdots + bk_p s + bk_i} \tag{27}$$

By setting (27) equal to the desired transfer function, $\omega_c^{n+1} / (s+\omega_c)^{n+1}$, the controller gains are determined independent of the plant as functions of bandwidth.

$$\left\{ k_j = \binom{n+1}{n-j} \frac{\omega_c^{n-j}}{b} \right\} j = -1, 0, \cdots, n-1 \tag{28}$$

Interestingly, this is equivalent to (24) when $\bar{a}_j = 0$.

To show how much *f* is rejected, the following transfer function is also derived using superposition.

$$\frac{y(s)}{f(s)} = \frac{s}{(s+\omega_c)^{n+1}} \tag{29}$$

Clearly, all frequencies are attenuated, especially those beyond the neighborhood of ω_c.

Remarks

1. The integrator in C(s) increases the order of λ(s), placing an *s* in the numerator of (29) to create the desired robustness and disturbance rejection.
2. The new structure is tuned to achieve a desirable closed loop response and disturbance rejection simultaneously. In the PID structure, a compromise exists: the presence of controller zeros in the closed loop transfer function does not allow it and λ(s) to be tuned at the same time.

Example 2

A method for controlling plants with finite zeros is now proposed. Consider the plant

$$G(s) = \frac{y(s)}{u(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \tag{30}$$

where $m \le n$. By cross multiplying, integrating *m* times, and collecting terms, the plant is transformed into the form

$$s^{\bar{n}} y(s) = f(s) + b_m u(s) \tag{31}$$

where

$$\begin{aligned} f(s) &= \left(b_{m-1}s^{-1} + b_{m-2}s^{-2} + \ldots + b_0 s^{-m}\right) u(s) \\ &- \left(a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_0\right) s^{-m} y(s) \end{aligned} \tag{32}$$

and $\bar{n} = n - m$ is denoted as the relative order of the system. It is considered an $\bar{n}$ integrator system with two inputs; an instantaneous input through $b_m u$ and a slower input disturbance through *f*, responding dynamically because it is a function of the output and integrated input.

$$y^{(\bar{n})} = f + b_m u \tag{33}$$

At this point, the application of model-independent techniques is straightforward. Control is applied by assuming the system is of relative order $\bar{n}$ with a relative gain $b = b_m$, the high frequency gain.

C. Output Differentiation

When the differentiation of the output is unavailable or too noisy in a particular application, a state observer is constructed from (33) using superposition

$$\begin{cases} \dot{z}_1 = z_2 + L_1(y - z_1) \\ \vdots \\ \dot{z}_{\bar{n}-1} = z_{\bar{n}} + L_{\bar{n}-1}(y - z_1) \\ \dot{z}_{\bar{n}} = L_{\bar{n}}(y - z_1) + b_0 u \end{cases} \tag{34}$$

where $z_1 \to y, \quad z_2 \to \dot{y}, \quad z_{\bar{n}} \to y^{(\bar{n})}$, and

$$\left\{ L_j = \binom{\bar{n}}{j} \omega_o^j \right\} j = 1, 2, \cdots, \bar{n} \tag{35}$$

The GIPD control law is easily modified to accept the observer's output, because it is already in state feedback form.

$$u = k_{-1} \int (r - y) - k_0 z_1 - \cdots - k_{r-1} z_{\bar{n}} \tag{36}$$

Example 3

Consider a plant with time delay.

$$G(s) = \frac{5s+2}{s^3+1} e^{-.05s}$$

The Smith Predictor is a widely accepted method of controlling a known plant with a known delay [16]. However, when the plant and delay are both unknown, the system is put into the canonical form given by (33).

$$\dddot{y} = f + 5u$$

Based on the system's relative order, a controller and observer are designed as

$$u = (\omega_c^3 \int (r - y) - 3\omega_c^2 z_1 - 3\omega_c z_2)/5$$

and

$$\begin{cases} \dot{z}_1 = z_2 + 2\omega_o(y - z_1) \\ \dot{z}_2 = \omega_o^2(y - z_1) + 5u \end{cases}.$$

The effectiveness of the new technique is illustrated in Fig. 6 for $\omega_c = 5$ and $\omega_o = 200$.

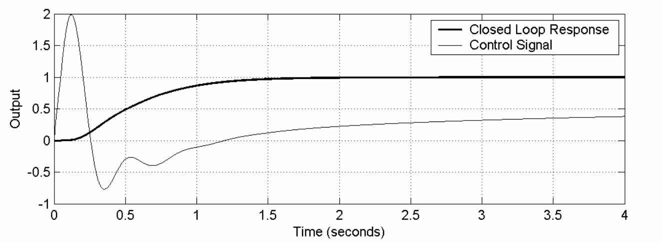

Figure 6. Controlling a System with Time Delay and Zeros

D. *Quality of the Control Signal*

In the PID structure, a step input causes a step in the control signal through the proportional gain. Higher order derivative gains make this abrupt change much worse. The generalized I-PD structure does not have this problem because the input is buffered by a single integrator in C(s) and higher order gains in H(s) do not suddenly affect the control signal when there is no sudden change in the output. The result is a smoother control signal with a smaller range, allowing an easier stay within saturation limits.

IV. SIMULATION

A nonlinear model of an actual "pick and place" application from the metal forming industry is used to demonstrate GIPD in a realistic simulation. In [8] this model is used to compare a handful of control techniques. The results show that ADRC outperforms the others in a number of ways. This being the case, the results here are compared with ADRC and discussed.

A. *A Realistic Motion Control Problem*

The servo amplifier/motor/transmission combination is modeled as

$$\begin{aligned} V_m &= K_{pa}(V_c - K_{cf} I_a) \\ I_a &= \frac{1}{R_a}\left(V_m - \frac{K_e}{R_p}\dot{x}\right) \\ \ddot{x} &= \frac{R_p}{J_t}(T_d + K_t I_a - T_f) \end{aligned} \tag{37}$$

where all of the variables are defined below.

Electical	
Control Input:	$V_c = \pm 8$ volts
Power Amplifier Gain:	$K_{pa} = 80$
Motor Voltage:	$V_m = \pm 160$ volts
Current Feedback Gain:	$K_{cf} = 0.075$ volts/amp
Back emf Constant	$K_e = 1.49$
Armature Resistance:	$R_a = 0.4\Omega$
Armature Inductance:	$L_a = 8$ mH
Mechanical	
Torque Constant:	$K_t = 1.5$ Nm/amp
Motor Inertia:	$J_m = 0.00565$ kg-m^2
Total Inertia:	$J_t = 0.113$ kg-m^2
Transmission Ratio:	$R_p = 1.25$
Coefficient of Friction:	$K_f = 1$
Load Mass:	$m = 106.6$ kg
Torque Disturbance:	$T_d = 100$ Nm
Output Position:	$x = 0 – 20$ mm

One assumption is made; L_a is small enough with respect to R_a that it can be ignored. Without nonlinearities, the transfer function of the system becomes

$$\frac{x(s)}{V_c(s)} = \frac{K}{s(s+\omega_p)} \tag{38}$$

where

$$\begin{aligned} K &= \frac{K_t R_p K_{pa}}{\left(R_a + K_{pa}K_{cf}\right)J_t} \\ \omega_p &= \frac{K_t K_e}{\left(R_a + K_{pa}K_{cf}\right)J_t} \end{aligned} \tag{39}$$

To provide a realistic model though, a number of nonlinearities are added. Transmission backlash is modeled with a ±1.25 m/sec. dead-bandwidth on $\dot{x}$. A 50 Hz resonant mode is placed before $\ddot{x}$, just over one decade above the desired operating bandwidth. The loss of torque due to viscous friction is given as

$$T_f = \operatorname{sgn}(\dot{x}K_f / R_p) + \dot{x}K_f / R_p\,. \tag{40}$$

Amplifier saturation is modeled by limiting $|V_c| \le 8$ volts and $|V_m| \le 160$ volts. Furthermore, 1% white noise is injected into the output and a large torque disturbance T_d = 100 Nm is applied at time t = 1 second.

B. *The Application of GIPD and ADRC*

Based on the order of (38), a GIPD control law and observer are selected to control the motion system with

$$u = (\omega_c^3 \int (r-y) - 3\omega_c^2 z_1 - 3\omega_c z_2)/b$$

and

$$\begin{cases} \dot{z}_1 = z_2 + 2\omega_o(y - z_1) \\ \dot{z}_2 = \omega_o^2(y - z_1) + bu \end{cases}.$$

The controller is then digitized with a sample rate of 1kHz using backward Euler integration. The gain b = 177 is roughly estimated instead of using (39), as is the case in a real application where all numbers are not given. With a step input, the controller bandwidth is tuned to produce the fastest response without overshoot. The observer bandwidth is then tuned to limit the control signal noise to about three volts. Here, $\omega_c = 30$ and $\omega_o = 300$.

The process is repeated by implementing the ADRC control law

$$u = (\omega_c^2(r - z_1) - 2\omega_c z_2 - z_3)/b$$

and the extended state observer

$$\begin{cases} \dot{z}_1 = z_2 + 3\omega_o (y - z_1) \\ \dot{z}_2 = z_3 + 3\omega_o^2 (y - z_1) + bu \\ \dot{z}_3 = \omega_o^3 (y - z_1) \end{cases}$$

with $\omega_c = 20$ and $\omega_o = 200$. The reason for the shift in bandwidth from GIPD to ADRC is now discussed. In this paper, the desired response is always given in the form of $\omega^n / (s+\omega)^n$. Higher orders of n produce more lag in the response time. To compare the responses of different orders, a rough form of normalization is suggested in Fig. 7 by increasing ω to $n\omega$ in each case.

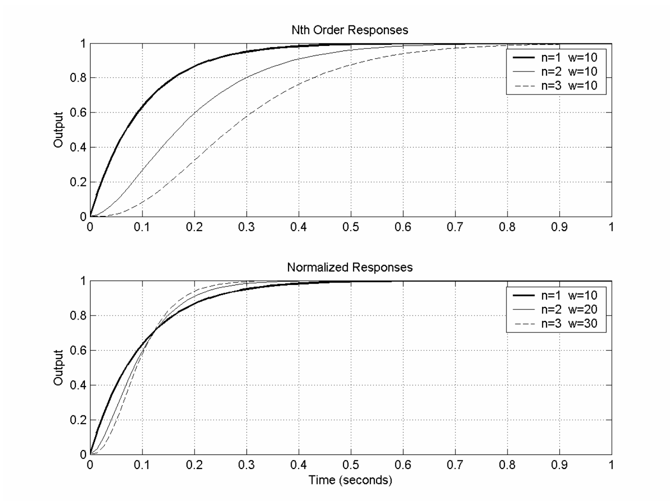

Figure 7. Normalizing Responses by Rise Time

Since GIPD produces a third order response and ADRC produces a second order response, the ADRC bandwidths are set to 2/3 that of GIPD. Fig. 8 shows results of both controllers. They are virtually identical, verifying the GIPD and normalization techniques.

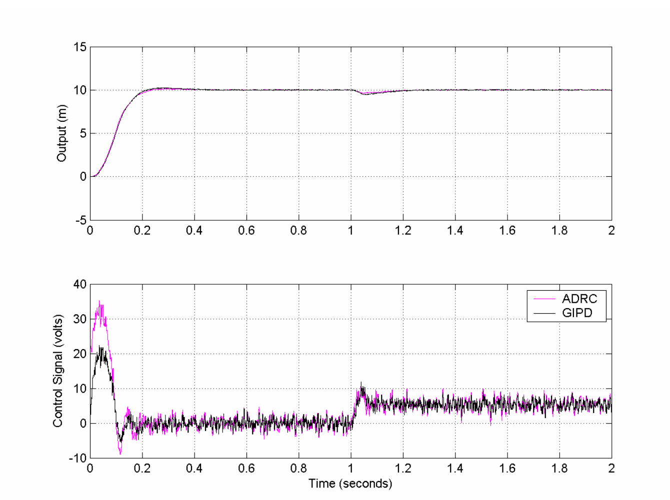

Figure 8. Comparing GIPD and ADRC in Motion Control

Remarks

1. When $\omega_c = \omega_o$, both loop gains are identical and reduce to

$$L(s) = \frac{\omega^3 (10s^2 + 5\omega s + \omega^2)}{bs(s^2 + 5\omega s + 10\omega^2)} G(s). \tag{41}$$

2. In GIPD, disturbances are cancelled with K_i by increasing ω_c. In ADRC, they are cancelled with z_3 by increasing ω_o.

C. A Practical Motion Profile

A motion profile is often used in industry to keep the transient error in a small range, preventing saturation and integrator windup. To remain practical, a motion profile with only two parameters is used

$$r = \begin{cases} r_{sp}\left(10\left(\frac{t}{t_s}\right)^3 - 15\left(\frac{t}{t_s}\right)^4 + 6\left(\frac{t}{t_s}\right)^5\right), \; 0 \le t < t_s \\ r_{sp}, \; otherwise \end{cases} \tag{42}$$

where the set point r_{sp} and the settling time t_s are specified.

This polynomial profile produces smooth position, velocity and acceleration trajectories while maintaining finite jerk. By setting the derivative equal to zero and taking the roots, the maximum value of each trajectory is calculated.

$$\begin{aligned} \dot{r}_{\max} &= 1.875\; r_{sp} / t_s \\ \ddot{r}_{\max} &= 5.7735\; r_{sp} / t_s^2 \, . \\ \dddot{r}_{\max} &= 60\; r_{sp} / t_s^3 \end{aligned} \tag{43}$$

An example of the profile's smooth output is shown below.

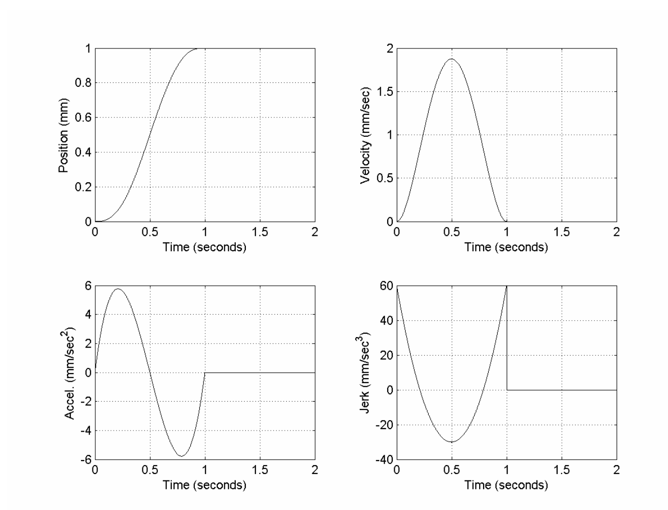

Figure 9. Polynomial Profile Trajectories

When the profile is applied to the motion control problem, Fig. 10 shows the control signal in the transient region is cut in half for both structures. This allows the controller bandwidths to be increased.

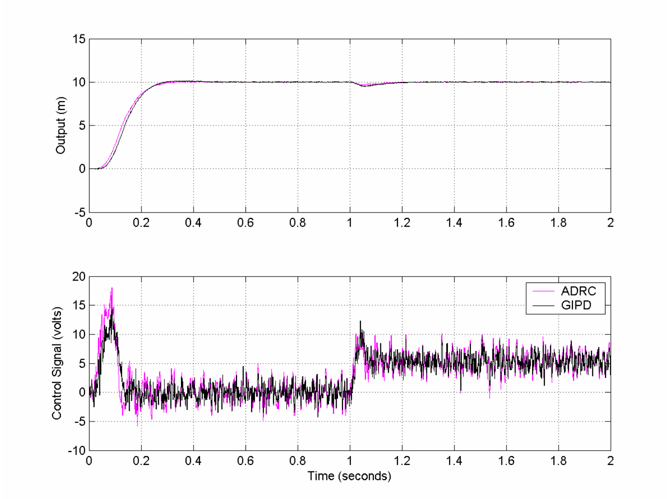

Figure 10. Motion Control Using a Polynomial Profile

V. CONCLUDING REMARKS

Although ADRC is fairly new, it has been implemented in many benchmark control problems. The underlying concepts involved are new to conventional thinking, yet they are powerful and show promise. This paper outlines a few model-independent tools that practicing engineers can find very useful in many applications. The motion control problem may be complex with many parameters, yet GIPD is a viable solution that can achieve high performance without knowledge of any of the system's parameters.

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