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From Poncelet's Invariance Principle to Active Disturbance Rejection

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Abstract— **This is a brief survey of a little known field of disturbance estimation and subsequent cancellation, a field with a long history and is still rather disorganized. Researchers and results are scattered over almost two centuries, across East and West: from Jean-Victor Poncelet's Principle of Invariance in 1829, to Jingqing Han's conception of Active Disturbance Rejection in 1995 and beyond. But the field in recent years is maturing and coming into a focus with significant practical and theoretical implications abound. It provides a powerful alternative to the modern control paradigm in how real world control problems, of which disturbance rejection is a central theme, are viewed and solved. In this paper a reader will find a brief history of ideas, a new, unifying, problem formulation, and a summary of recent stability analysis results.**

I. INTRODUCTION

This paper concerns with a fundamental question of feedback control system design: how do we best deal with disturbances in a process and achieve a consistent performance in a system consists of inconsistent parts, operating in an environment full of unknowns. In a feedback control system, the input to a plant is manipulated by the controller so that its output is what we desire. In addition to the input, however, this plant output is also affected by disturbances. Therefore, making a process insensitive to such disturbances is a task that is central to any control system design, and it is known as disturbance rejection.

It may come as a surprise that, important as it is, disturbance rejection has been carried out, for the most part any way, rather passively, in both theory and practice. In modern control proper, disturbance is among many competing concerns in design, to which the system response is shaped. The notion of disturbance rejection is mostly synonymous to disturbance attenuation. Another body of theoretical research is rooted in the well known internal model principle, as nicely summarized in [1], where the disturbance rejection is achieved, in the presence of plant parameter variations, if the mathematical model of the disturbance is given and incorporated into the controller. On the practical side, the method of choice, proportional-integral-derivative (PID) control, is just as indirect and passive. As disturbance works its way through a process, it eventually leads to changes in the process output, causing it to deviate from the setpoint. And PID simply reacts to such deviations as they occur. The question is how effective such disturbance rejection can be, given that by the time controller reacts to it, the damage may already be done, sometimes irrevocably. The answer to such a question is important, of course, and it may have wide practical implications. Unfortunately, apart from a few scattered reports stretching the last seven decades, little has been done to systematically investigate alternatives in disturbance rejection and to evaluate the accumulative research results in their totality.

This paper provides a short survey of one such alternative, the idea of active disturbance rejection that stretches from the 19th Century to the present. It is a simple idea of seeking out the disturbance and cancelling it out with the control action before it does damage. It has the simplicity and elegance of reducing a complex process to a simple, disturbance-free, plant easily controlled by an "invariant" controller, which is made possible by actively measuring or estimating the disturbance and counteracting it with the control action. It is a rather different paradigm, as contrasted to the modern mathematical tradition in control science. It is not about closing the loop and using feedback to change the dynamics of the process from what is given to what is desired. It is about taking out what's undesired from the process before the feedback is applied. Practically speaking, active disturbance rejection may very well provide the elusive answer to the question: how do we accommodate large amount of uncertainties commonly seen in industrial controls?

Although the results are far and few in between, upon closer examination, we did find a trail of ideas in the literature as generations of researchers and engineers sought to tackle the above question. And in this paper we attempt to give an overview of and explore the interconnection among their ideas, design techniques and the more recent results of stability analysis, through which we seek a better understanding of disturbance rejection problem itself and, perhaps, a new way of thinking.

The paper is organized as follows: a brief history of ideas is given in section II, followed by a new formulation of disturbance rejection problems that leads to a unified framework upon which all past, seemingly unrelated, results can be examined, with their connections exposed. The stability analysis results are summarized in the ensuing section, leading to concluding remarks in section IV.

II. A BRIEF HISTORY OF IDEAS

In the process of improving the control system performance for steam engine, French engineer/mathematician Jean-Victor Poncelet seems to be the first person to suggest, in 1829, that disturbance be "used to generate an activating signal which will tend to cancel the effect of the same

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disturbance"[2]. This idea is later known as the invariance principle, which was formally established in the then Soviet Union by Gheorghe Vladimirovich Shipanov in 1939 [3]. Although the invariance principle becomes classics in control theory in Soviet Union [4], it did not appear to generate much interest in the West. Interestingly, Shipanov's theory impressed upon the young mind of a Chinese doctoral student, Jingqing Han, who studied in Moscow State University in 1960s and who would revive the idea three decades later [5].

In the meanwhile, apparently unaware of Shipanov's earlier work, Johnson proposed in 1971 [6] to treat input disturbance as a fictitious state, to be estimated with a state observer, known later on as the Unknown Input Disturbance Observer (UIO), and canceled directly with the control signal. Taking advantage of the state space method in modern control theory of the 1960s, UIO utilizes the input and output data from the plant and estimate the disturbance as well as the state, assuming that the plant and disturbance dynamics is known.

Unaware of UIO, Japanese researchers developed a similar input disturbance observer in 1987 using the transfer function approach known simply as Disturbance Observer (DOB) [7]. A discrete realization of DOB, denoted as Perturbation Observer, was also proposed [8]-[9]. Schrijver and Dijk established the equivalence between UIO and DOB [10]. But it was not until 2006 that disturbance observers as a class by itself was surveyed and brought into the large family of observers [11], thus bringing the long overdue attentions to this field, see for example [12]. By and large, these disturbance observers are different manifestations of the invariance principle but the researchers were apparently unaware of it and they are rather concerned with the solutions of individual engineering problems, not the underlying scientific principles. Consequently, important as it is, this field of research is still rather disjoint and obscure.

Uncertainties a control system must contend with come from both unmodeled dynamics and external disturbance, and they are handled separately in the modern control paradigm with the former falls under robust control. All above disturbance observers share a common premise: the mathematical model of plant dynamics is accurately known, even though some researchers speculated that perhaps to some degree modeling inaccuracy can be tolerated, if not addressed [9], [10]. From engineering perspective, this still leaves the problem of uncertainty largely unresolved, particularly those large uncertainties that can not be brought into the robust control framework constrained by the Small Gain theorem.

Tornambe and Valigi proposed in 1994 to robustly stabilize SISO systems, in the context of vehicle stability control, by estimating the unmodeled dynamics in a way similar to disturbance estimation [13]. To distinguish this from the disturbance observers, we denote it as the Dynamics Estimator (DES). A significant constraint in DES is that it requires all states to be measured before the unknown dynamics is estimated and canceled like a disturbance.

Finally in 1995, Han proposed a single solution to deal with both the dynamic uncertainties and the unknown external disturbances: the Extended State Observer (ESO), by which the combined effect of both unknown dynamics and external disturbance is treated as a fictitious state, estimated using a state observer, and canceled out, reducing a complex nonlinear time-varying control problem to a simple linear time-invariant one [5]. Note that Han expanded the notion of disturbance to include both unmodeled dynamics and external disturbance and, to distinguish it from a mere external disturbance, we denote it as *total disturbance* and Han's idea as Total Disturbance Estimation. The control design based on ESO actively seeks out *total disturbance* and cancel it out, forming the backbone of a new design paradigm: Active Disturbance Rejection Control (ADRC) [5], [14], [15]. (Some translations had it as auto-disturbance rejection control,until Han's ideas were systematically introduced into the English literature first in 2001 [16], and then in 2009 [17].) ESO was compared to DOB, sliding mode observer, and high gain observer [18], [19].

III. Total Disturbance Estimation and Active Disturbance Rejection

Consider a general nonlinear single-input single-output (SISO) plant in the state space form as

$$\begin{aligned}\dot{x} &= A(x) + B(x)\left(a(x,z,d) + b(x,z,d)u\right) \\ \dot{z} &= g(x,z,d) \\ y &= C(x)\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n, z \in \mathbb{R}^m, u \in \mathbb{R}, y \in \mathbb{R}$ are observable state, unobservable state, input and output of the system, respectively, $d \in \mathbb{R}^p$ is external input disturbance, $A(\cdot), B(\cdot), C(\cdot)$ are known nonlinear functions, $g(\cdot)$ is an unknown nonlinear function, $a(\cdot)$ and $b(\cdot)$ are unknown or at most partially known nonlinear functions. The unobservable dynamics is assumed to be bounded-input-bounded-state (BIBS) stable. Such description restricts the unknown dynamics and disturbances to be directly associated with the control input u. In contrast, most control design techniques used today assume a disturbance free plant such as

$$\begin{aligned}\dot{x} &= A(x)x + B(x)\left(\hat{a}(x) + \hat{b}(x)u\right) \\ y &= C(x)\end{aligned} \tag{2}$$

where $A(\cdot)$, $B(\cdot)$, $C(\cdot)$, $\hat{a}(x)$ and $\hat{b}(x)$ are known nonlinear functions. Assuming that the control law designed for (2) is

$$u = \hat{b}(x)^{-1}\left(K(x) - \hat{a}(x)\right) \tag{3}$$

the question is to what degree the solution for (2) can be applied to the original nonlinear plant in (1) if $\hat{a}(x)$ and $\hat{b}(x)$ do not closely approximate $a(x,z,d)$ and $b(x,z,d)$, respectively?

A. Total Disturbance Rejection

We are particularly interested in the cases with significant uncertainties, either in the dynamics of the plant or in the external disturbances, or both, where such close approximations do not hold. It is here that we introduce the notion of the *total disturbance*.

The plant in (1) can be rewritten as

$$\begin{aligned} \dot{x} &= A(x) + B(x)\left(\hat{a}(x) + \hat{b}(x)(u+f)\right) \\ y &= C(x) \end{aligned} \tag{4}$$

where f, denoted as the *total disturbance*, represents both the unknown dynamics that is internal to the plant and the external disturbances, can be obtained as

$$f = \hat{b}(x)^{-1}\left(a(x,z,d) - \hat{a}(x) + (b(x,z,d) - \hat{b}(x))u\right) \tag{5}$$

Now, if we take a leap of faith and treat f as a signal that can be estimated from the input-output data of the plant, then the closed-loop system will remain unaffected by the *total disturbance* with a simple modification of the control law in the form of

$$u = \hat{b}(x)^{-1}\left(K(x) - \hat{a}(x)\right) - \hat{f} \tag{6}$$

where $\hat{f}$ is the real time estimate of f. Almost like a magic, this idea works, as seen in one engineering application after another. Recently theoretical analysis is also backing it up, as shown later in the paper.

We take a moment to appreciate the importance of Han's work. He discovered, in 1995, that the internal disturbance (uncertain dynamics) and external disturbances can be rejected all together in a simple and elegant way. He made possible a new understanding that control problems are essentially disturbance rejection problems and the disturbances are best actively rejected instead of passively accommodated. Furthermore, this new paradigmatic shift breaks down those artificial boundaries between linear and nonlinear systems, between time varying and time-invariant systems, and helps us to identify a new set of fundamental principles, which allows a new set of tools in dealing with the challenging problems that we encounter in the real world.

B. Total Disturbance Estimation

To implement the solution (6), both x and f need to be estimated, since x is generally not available for feedback. To this end, Han proposed the ESO as the solution by making f an extended state variable, rewriting the state equations, and building a state observer. For the sake of clarity, we denote the ESO as it is applied to the plant (4) TDE, as shown in Figure 1.

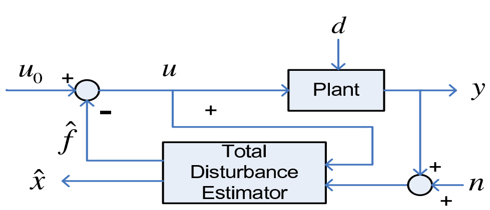

Fig. 1. Estimation and Rejection of Total Disturbance

Given the nominal information $A(x), B(x), C(x), \hat{a}(x)$ and $\hat{b}(x)$, a TDE is constructed as follows

$$\begin{aligned} \dot{\hat{x}} &= A(\hat{x}) + B(\hat{x})\left[\hat{a}(\hat{x}) + \hat{b}(\hat{x})\left(u + \hat{f}\right)\right] \\ &\quad + L_1\left(y_m - C(\hat{x})\right) \\ \dot{\hat{f}} &= \hat{b}(\hat{x})^{-1} L_2\left(y_m - C(\hat{x})\right) \end{aligned} \tag{7}$$

where L_1 and L_2 are observer gains to be selected. (Multiple extended states can be constructed to estimate f and its derivatives. Correspondingly, L_2 may be a vector. For simplicity, however, in this paper only one disturbance state is used). ESO as it was originally proposed employs nonlinear gains. Selections and parameterization of such gains were addressed by Gao in [20]. Corresponding to (6), u_0 in Figure 1 is given as

$$u_0 = \hat{b}(\hat{x})^{-1}\left(K(\hat{x}) - \hat{a}(\hat{x})\right) \tag{8}$$

Note that all previous disturbance observers, including UIO, DOB(POB), and DES can be formulated in the TDE form. For UIO, this demonstrates that it not only estimates the external disturbance, but also the unknown internal dynamics; for DES, on the contrary, this illustrates that it not only estimates the unknown internal dynamics, but also the external disturbance. For DOB, although it is equivalent to UIO in estimating f, its transfer function implementation has the disadvantage of not allowing the state to be estimated at the same time, limiting the choice of the control laws. Regardless of the particular form of disturbance estimation, these different disturbance rejection methods find a unifying principle in ADRC, i.e. the disturbance is actively rejected as opposed to passively accommodated. For this reason, they can all be viewed as a particular implementation of ADRC.

If we treat (4) with $f = 0$ as the *nominal model* of the plant, then TDE estimates whatever difference between it and the actual plant. By estimating f and cancelling it out, we allow the controller to be invariant in the presence of uncertain dynamics and external disturbance and this is the remarkable strength of active disturbance rejection.

IV. Stability Analysis

In this section we provide a brief summary of stability analysis results for various disturbance observers. For convenience and clarity, they are presented in the TDE framework and characterized according to the model used, the forms of observer and controller, the methods of stability analysis, the assumptions made, and the conclusions drawn. For uniformity, the symbols in original papers may be changed. Moreover, only the result for n^{th}-order plant are presented while less general work is briefly mentioned.

A. Stability Proof for External Disturbance Estimation and Rejection

UIO and DOB are both designed to estimate external disturbances, in state space and transfer function representations, respectively. In stability analysis of DOB, Bickel and Tomizuka derive the conditions for the input/output stability of a system representing a class of robotic manipulators and the asymptotic stability of the system without external disturbance [21]; DOB is further extended to nonlinear plants by Chen *et. al* and Back *et. al*, where stability robustness is also studied [22], [23].

A stability analysis of UIO applied in vehicle steering control is provided by Hahn *et. al* in [24]. UIO is also applied in Fault Detection and Isolation with actuator faults chosen as extra states, the dynamics of which are usually called adaptive control law or adaptive algorithm in the adaptive control context [25]. For example, Demetriou applies UIO to 2^{nd}-order systems and provides stability analysis [26]. Recently, Wang and Lum Apply multiple parallel UIOs, corresponding to different models under the combination of actuator faults, to determine the aircraft actuator fault location and give a general-form stability analysis [27]. For simplicity, only one of the three actuator fault types studied in [27], the *Lock in place* , is presented here with the simplification of the plant.

1) Plant Model/Nominal Model:

In the sense of disturbance estimation, suppose i^{th} actuator is locked, the system described in [27] is

$$\begin{aligned} \dot{x} &= Ax + B_i u + b_i \bar{u} + Ed \\ y &= Cx \end{aligned} \tag{9}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^r$, $d \in \mathbb{R}^q$, the vector $u \in \mathbb{R}^p$ represents p actuators, $\bar{u} \in \mathbb{R}$ is the unknown constant lock value of actuator i, A, B, C, E are known matrices representing the plant without fault, B_i is B with i^{th} column all zero, and b_i denotes the i^{th} column of B.

The *nominal model* of the system is

$$\begin{aligned} \dot{x} &= Ax + B_i u \\ y &= Cx \end{aligned} \tag{10}$$

2) Disturbance Definition:

The unknown input is defined as

$$f(u) = \bar{u} \tag{11}$$

Note that the external disturbance d is not included in f because the matrices selection $(I - HC)E = 0$ makes the disturbance term cancelled in the estimation error dynamics of the UIO in the following discussion, since A, B, C and E are exactly known.

3) Observer/Controller:

With an augmented state $\hat{f}$ to estimate the unknown lock value of actuator i, a UIO is established as follows.

$$\begin{aligned} \dot{w} &= Fw + GB_i u + Gb_i \hat{f} + Ky_m \\ \dot{\hat{f}} &= L_2(y_m - C\hat{x}) \\ \hat{x} &= w + Hy_m \end{aligned} \tag{12}$$

where w and $\hat{x}$ are UIO state vector and plant state estimation vector respectively, L_2 is a gain matrix to be designed with adaptive control method, and matrices F, G, K and H are determined to make the UIO stable: $(I - HC)E = 0, G = I - HC, F = A - HCA - K_1 C, K_2 = FH, K = K_1 + K_2$, and F is Hurwitz.

4) Stability Analysis:

Define $e = x - \hat{x}$ and $\Delta f = \bar{u} - \hat{f}$. After derivation, the UIO estimation error dynamics is obtained as

$$\begin{aligned} \dot{e} &= Fe + Gb_i \Delta f \\ \Delta \dot{f} &= L_2 Ce + \dot{\bar{u}} \end{aligned} \tag{13}$$

Since $\bar{u}$ is a constant during the faulted period, the stability of the system depends on the gain matrix L_2, which needs to make $\begin{bmatrix} F & Gb_i \\ L_2 & 0 \end{bmatrix}$ Hurwitz. The Lyapunov's direct method is then used to determine the stability condition. Wang *et al.* establish that the globally exponentially stability for the UIO estimation error dynamics is guaranteed if

- The matrix A is Hurwitz.
- The estimation error of the locked position, $\Delta f = f - \hat{f}$, is bounded and globally Lipschitz.
- The estimation error Δf is persistently exciting (PE).

B. Stability Proof for Model Uncertainty Estimation and Rejection

Chakrabortty and Arcak provide stability analysis for a controller designed to estimate uncertain internal dynamics in 2006 [28]. Actually, back to 1994, in the original DES paper, Tornambe and Valigi also provide stability analysis for a more general plant, presented as below[13].

1) PLant Model/ Nominal Model:

Consider a system

$$\begin{aligned} \dot{x} &= Ax + B\left(a(x,z) + b(x,z)u\right) \\ \dot{z} &= g(x,z) \\ y &= Cx \end{aligned} \tag{14}$$

where $x \in \mathbb{R}^n, z \in \mathbb{R}^m, y \in \mathbb{R}, u \in \mathbb{R}$, and A, B, C represents a n^{th}-order cascaded integral plant.

The *nominal model* of the system is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{15}$$

2) Disturbance Definition:

The unknown input is defined as the discrepancy between the nonlinear plant and its nominal model:

$$f(x,z,u) = \dot{x}_n - u = a(x,z) + \left(b(x,z) - 1\right) u \tag{16}$$

3) Observer/Controller:

Assuming $x_1, \cdots, x_n$ are available, a DES is established as follows.

$$\begin{aligned} \dot{\xi} &= -\beta_{n-1}\xi - \sum_{i=0}^{n-2} \beta_i x_{i+2} \\ &\quad -\beta_{n-1}\left(\sum_{i=0}^{n-1} \beta_i x_{i-1} + u\right) \qquad (17) \\ \hat{f} &= \xi + \sum_{i=0}^{n-1} \beta_i x_{i+1} \end{aligned}$$

where ξ and $\hat{f}$ are DES state vector and model discrepancy estimation respectively, $\beta_{n-1} := \sigma\left(b(x,z)\right)\mu$ (μ is a suitable positive constant and $\sigma(x)$ is the sign of x), and $\beta_i, i = 0, \cdots, n-2$ are arbitrary constants.

Eq. (17) is rather complex but it can similarly obtained by simply passing the model discrepancy $f(x, z, u) = \dot{x}_n - u$ through a low pass filter of the form

$$\dot{\hat{f}} = \beta_{n-1}(f(x, z, u) - \hat{f}) \quad (18)$$

The idea of this DES is manipulating (18) with state variable substitution so that the required available highest order derivative of the output is $y^{(n-1)}$ instead of $y^{(n)}$.

With the model discrepancy estimated and rejected by

$$u = u_0 - \hat{f} \quad (19)$$

and the forced n^{th}-order cascaded integral plant is regulated by

$$u_0 = -Kx \quad (20)$$

where K is the controller gain vector to make the n^{th}-order cascaded integral plant Hurwitz.

4) Stability Analysis:

The Lyapunov's direct method is employed to prove the stability of the DES system. Because there is no external input to the system by assumption, the asymptotic stability is established with a lower bound of μ based on the following assumptions.

- There exists a radially unbounded Lyapunov function $V(x, z)$ such that 1) $V(0, 0) = 0$, $\frac{\partial V}{\partial \zeta}|_{\zeta=0} = 0$; 2) $\frac{\partial V}{\partial x}(A - BK)x + \frac{\partial V}{\partial z} g(x, z) \leq -\|\zeta\|^2$ for the defined compact domain, where $\zeta = [x^T z^T]^T$.
- The function $b(x, z)$ is continuous, always positive or negative and norm-lower-bounded by a positive constant $\underline{b}$ for the domain of interest
- The sign of b is known for the domain of interest.

C. Stability Proof for Total Disturbance Estimation and Rejection (1)

Only the ESO-based control algorithms are designed with the purpose of *total disturbance* estimation and rejection. Zhou *et. al* formulate an n^{th}-order system with ADRC controller as a singular perturbation problem and applies variation of Lyapunov's direct method to prove the exponential stability of the system based on several assumptions [29]. In another paper on ADRC stability, Zheng *et. al* provide a less conservative stability analysis [30], which is summarized below.

1) Plant Model/Nominal Model:

Consider a system

$$\begin{aligned} \dot{x} &= Ax + B\left(a(x, d) + bu\right) \\ y &= Cx \end{aligned} \quad (21)$$

where $x \in \mathbb{R}^n, y \in \mathbb{R}, u \in \mathbb{R}, d \in \mathbb{R}$, A, B, C represent a plant of n cascaded integrators, $a(\cdot)$ is an unknown nonlinear function, and b is an unknown constant.

The *nominal model* of the system is

$$\begin{aligned} \dot{x} &= Ax + B\hat{b}u \\ y &= Cx \end{aligned} \quad (22)$$

where $\hat{b}$ is the nominal value of b.

2) Disturbance Definition:

The *total disturbance* to be estimated is

$$f(x, d, u) = \hat{b}^{-1}\left(a(x, d) + (b - \hat{b})u\right) \quad (23)$$

Based on the fact that b can be determined accurately in some application, $\hat{b}$ is assumed to be equal to b. Thus the *total disturbance* is

$$f(x, d) = b^{-1}a(x, d) \quad (24)$$

3) Observer/Controller:

With an extended state defined as $x_{n+1} = bf(x, d)$, the system (21) can also be written in augmented state space form:

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{B}bu + E\dot{f} \\ y &= \bar{C}x \end{aligned} \quad (25)$$

where $\bar{A} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $E = \begin{bmatrix} 0 \\ B \end{bmatrix}$, and $\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$

Let states $x_1, \cdots, \hat{x}_{n+1}$ estimate $x_1, \cdots, x_{n+1}$ respectively, an ESO is constructed:

$$\dot{\hat{x}} = \bar{A}\hat{x} + \bar{B}bu + L(y_m - \bar{C}\hat{x}) \quad (26)$$

where $L = [\begin{smallmatrix} L_1 \\ L_2 \end{smallmatrix}]$ is the ESO gain vector, and the *total disturbance* estimate is $\hat{f} = b^{-1}x_{n+1}$. (Note here the augmented state space form of the ESO is equivalent to the form in (7))

The *total disturbance* is rejected by (19) and then the forced nominal plant is stabilized by

$$u_0 = b^{-1}\left(-K\hat{x}\right) \quad (27)$$

4) Stability Analysis:

By applying Lyapunov's direct method, Zheng *et. al* establish the asymptotic stability condition for both the ESO estimation error and the system tracking error based on the following assumptions

- $f(x, d)$ is given.
- $\dot{f}(x, d)$ is globally Lipschitz with respect to x.

By solving the state space equations, Zheng *et. al* also establish the boundedness of the ESO estimation error and the system tracking error without the above assumptions. Furthermore, it is established that the upper bounds of both errors are functions of ω_o and ω_c assuming

- $\dot{f}(x, d)$ is bounded.

D. Stability Proof for Total Disturbance Estimation and Rejection (2)

The latest stability analysis for *total disturbance* estimation and rejection is provided by Freidovich and Khalil [12]. In this work the ESO is applied in the framework of the Nonlinear High-gain Observer (NHO) [31] to form an ENHO.

1) Plant Model/Nominal Model:

$$\begin{aligned}\dot{x} &= Ax + B\left(a\left(x,z,d\right) + b\left(x,z,d\right)u\right) \\ \dot{z} &= g\left(x,z,d\right) \\ y &= Cx\end{aligned} \tag{28}$$

where $x \in \mathbb{R}^n, z \in \mathbb{R}^m, u \in \mathbb{R}, y \in \mathbb{R}, d \in \mathbb{R}^p$, and A, B, C represent a plant of n cascaded integrators (in [12], a and b are exchanged).

The nominal model of the system is

$$\begin{aligned}\dot{x} &= Ax + B\left(\hat{a}(x) + \hat{b}(x)u\right) \\ y &= Cx\end{aligned} \tag{29}$$

2) Disturbance Definition:

The definition of the extended state in [12] can be seen in the conclusion section: perturbation due to model uncertainty and disturbance. According to the notion of TDE, this extended state is equivalent to the *total disturbance*:

$$f(x,z,d,u) = \hat{b}(x)^{-1}\left(\Delta a + \Delta b u\right) \tag{30}$$

where $\Delta a = a(x,z,d) - \hat{a}(x)$, $\Delta b = b(x,z,d) - \hat{b}(x)$. (in [12] the extended state $\sigma = \hat{b}^{-1}f$)

3) Observer/Controller:

The ENHO in TDE form is

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + B\left[\hat{a}(\hat{x}) + \hat{b}(\hat{x})(u + \hat{f})\right] \\ &\quad + L_1(y_m - C\hat{x}) \\ \dot{\hat{f}} &= \hat{b}(\hat{x})^{-1}L_2(y_m - C\hat{x})\end{aligned} \tag{31}$$

where $\hat{a}, \hat{b}$ are approximations of a, b, and L_1 and L_2 are chosen to make eigenvalues of the observer in LHP.

The total disturbance is rejected by (19) and then the forced nominal plant is stabilized by

$$u_0 = \hat{b}(\hat{x})^{-1}\left(-K\hat{x} - \hat{a}(\hat{x})\right) \tag{32}$$

4) Stability Analysis:

The plant and the observer error dynamics are formulated as a singularly perturbed system. By applying variation of Lyapunov's direct method, Freidovich and Khalil establish the BIBS stability of the system with an lower bound of ω_o based on the following five assumptions.

- The vector d belongs to a known compact set and its derivative is bounded.
- a, b are continuously differentiable and their derivatives are locally Lipschitz; $b \geq b_0$ with a known $b_0 > 0$, and g is locally Lipschitz.
- The unobservable state dynamics in (28) is BIBS stable (in original NHO paper [31], this part represents the zeros of the system).
- The infinity norm $k_b = \max\limits_{(x,z)\in\Omega_c, d\in D}\left|\frac{\Delta b}{\hat{b}(x)}\right| < \frac{1}{\|G(s)\|_\infty}$, where $G(s) = \frac{\alpha_{n+1}}{s^{n+1}+\alpha_1 s^n+\cdots+\alpha_{n+1}}$.
- The initial states of the system and the observer are inside the defined compact set.

The conclusion that the system error norm approaches 0 as $\omega_o \to \infty$ is then established. Freidovich and Khalil also give the conditions for the asymptotic stability of the system when d is a constant.

E. Stability Analysis Comparison

Although presented in the same framework of TDE, the stability analyses and conclusions of the disturbance-observer-based control approaches briefly summarized in this section are quite different from and not well connected to each other. This is partly due to the fact that researchers have not recognized that there is a common principle of TDE that ties all their work together and they, for the most part until very recently, worked mostly in isolation. And this makes it rather a challenging task to present different work from the past cohesively. Nonetheless, we try to highlight the unique features of stability analysis carried out under various names, such as UIO, DES, ESO, and ENHO. To this end, a brief summary is shown in Table I, where we try to differentiate each study with regard to 1) the type of plant considered, 2) a priori information needed for analysis, 3) the assumptions made, and 4) conclusions drawn, as listed in the left column of the table.

In summary, the stability analyses for UIO and DES presented above focus on the constant input disturbance rejection and internal unknown dynamics rejection, respectively. Their exponential and asymptotic stability assessments are obtained with the assumptions of no unknown internal dynamics and no external disturbance, respectively, which makes their results applicable to a rather narrow range of problems.

The stability analysis carried out in ESO and ENHO concerns with the *total disturbance* and is more general. Between them, the ESO stability analysis focuses on the practical aspects (assuming, as the case in the real world, the parameter b is normally a constant with a small amount of uncertainties), and obtains the stability condition premised upon the boundedness of $\dot{f}$, which is again a reasonable assumption in physical systems. Furthermore, the study establishes a clear connection between the bandwidth of ESO and the upper bound of the estimation error, which helps the users to tune observer in practice. The ENHO stability, on the other hand, concerns with a more general scenario where $b(\cdot)$ is an nonlinear function and where there may exist unobservable states, with the trade-off that the stability constraints are perhaps a little more conservative and less intuitive.

TABLE I

MAIN RESULT SUMMARY OF THE DIFFERENT DISTURBANCE OBSERVERS

	UIO	DES	ESO	ENHO
Plant	linear	$d = 0$	b constant	-
$f(\cdot)$	$f(u)$	$f(x,z,u)$	$f(x,d)$	$f(x,z,d,u)$
Info needed	A, B, C, E	system order, $\hat{b}$	system order	$\hat{a}(\cdot), \hat{b}(\cdot)$, system order
Assump-tions	A Hurwitz; Δf is PE, bounded, Lipschitz	z is stable; $b(\cdot) \geq b_0$; $sign(b)$ is known	$\dot{f}(\cdot)$ is bounded	$\dot{d}$ bounded; $a^{(i)}, b^{(i)}$ smooth; z b.s
Conclu-tion	observer error g.e.s	system error a.s	system b.s	system b.s

a.: asymptotically, b.: BIBS, e.: exponentially, g.: globally, s: stable

V. CONCLUDING REMARKS

In this paper we traveled back in time and discovered what has happened in the history of control pertaining to disturbance estimation and rejection. We found a field of ups and downs, dating back to 1829. It is a field that is still quite disjoint in recent times with progress far and few in between and researchers often not aware of either past or current results in the same field. By providing a unifying framework in this paper, in the form of *total disturbance* estimation and active disturbance rejection, it is our hope that past results can be meaningfully brought together and surveyed, making it possible to find connections among each other, to distinguish redundancies from novelties, and to share the knowledge gained.

In the process, we believe that we come up on an alternative paradigm to address the pressing issues of control practice, particularly the ones involving large amount of uncertainties. Perhaps the *total disturbance* estimation and rejection framework formulated here could go a long way in helping practitioners thinking through the problems and finding solution and in helping theoreticians finding an exciting field of research that full of intriguing problems, such as how much uncertainty can be estimated and rejected based on the input-output data of a plant, under what conditions stability of such systems are assured, etc.

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