

V. Practical Optimization

- Scaling
- Practical Optimality
- Parameterization
- Optimization
 - Objectives
 - Means
 - Solutions
 - ω_c -optimization
 - Observer based design

Definition of Gain and Frequency Scales

$$G_p(s) \Rightarrow kG_p(s)$$

$$G_p(s) \Rightarrow G_p(s / \mathbf{w}_p)$$

$$\frac{23.2}{s(s+1.41)} = \frac{11.67}{\frac{s}{1.41} \left(\frac{s}{1.41} + 1 \right)} = \frac{k}{\mathbf{w}_p \left(\frac{s}{\mathbf{w}_p} + 1 \right)}$$

$$k = 11.67$$

$$\mathbf{w}_p = 1.41$$

Unit-Gain Unit-Bandwidth Plants

$$\frac{1}{s+1}, \frac{1}{s}, \frac{1}{s^2 + 2\mathbf{x}s + 1}, \frac{1}{s(s+1)}, \frac{1}{s^2}, \frac{1}{s^3 + \mathbf{x}_1 s^2 + \mathbf{x}_2 s + 1}, \dots$$

$$\frac{s+1}{s^2 + 2\mathbf{x}s + 1}, \frac{s^2 + 2\mathbf{x}_z s + 1}{s^3 + \mathbf{x}_1 s^2 + \mathbf{x}_2 s + 1}, \dots$$

- Controllers for UGUB plants are predetermined

Scaling of a controller for a new plant

- Stop reinventing the wheel
- A class of processes/plants, differ only in dc-gain and bandwidth
- Normalization of plants: unit-gain unit-bandwidth
- Controller design for UGUB plants
- Controller is obtained by scaling UGUB controllers

$$\{G_p(s), G_c(s)\} \Rightarrow \{kG_p(s/\omega_p), G_c(s/\omega_p)/k\}$$

Scaling of PID

$$\{G_p(s), G_c(s) = k_p + \frac{k_i}{s} + k_d s\}$$

⇓

$$\{kG_p(s/w_p), G_c(s) = (k_p + k_i \frac{w_p}{s} + k_d \frac{s}{w_p}) / k\}$$

⇓

$$\bar{k}_p = \frac{k_p}{k}, \bar{k}_i = \frac{k_i w_p}{k}, \bar{k}_d = \frac{k_d}{k w_p}$$

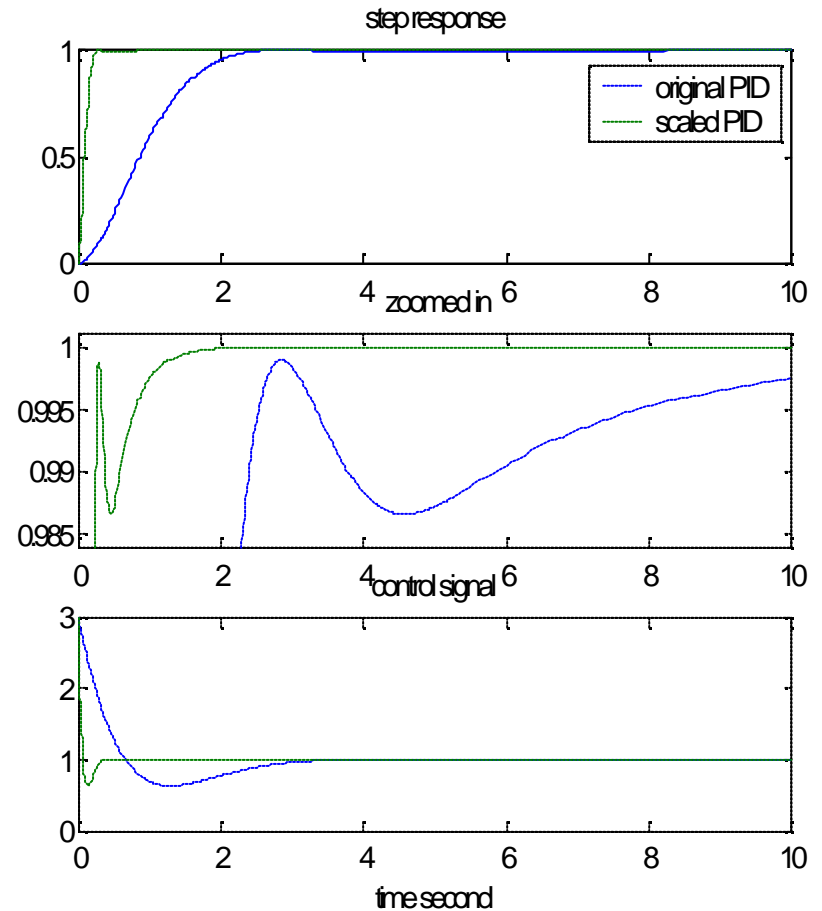
Scaling Example

$$G_p(s) = \frac{1}{s^2 + s + 1}$$

$$k_p=3, k_i=1, \text{ and } k_d = 2.$$

$$G_p(s) = \frac{1}{\left(\frac{s}{10}\right)^2 + \frac{s}{10} + 1}$$

$$\bar{k}_p = 3, \bar{k}_i = 10, \bar{k}_d = .2$$



Practical Optimality

Optimization of Control Law:

To maximizing a function of given performance measures subject to the physical limitations of the design

Quality of Control:

Speed, Accuracy,
Disturbance Rejection

Constraints:

Sampling Rate, Sensor
Noise, Uncertainty,
Actuation Smoothness,
Saturation

w_c -Optimization

maximizing the loop gain bandwidth, ω_c , subject to the physical limitations of the design.

w_c - Parameterization

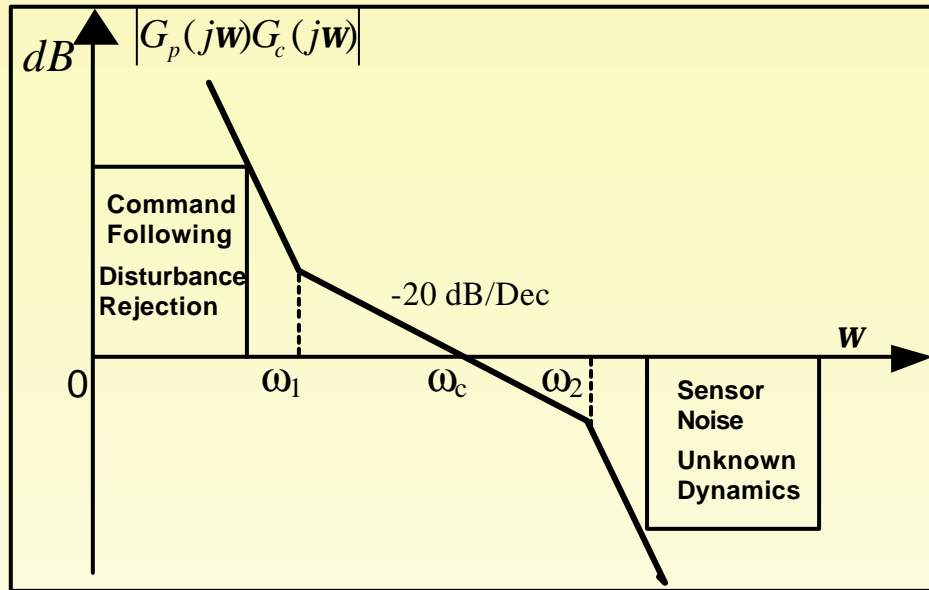
parameterization of the controllers, $G_c(s, \omega_c)$, for the unit gain and unit bandwidth that result in the loop gain bandwidth to be ω_c , or closed-loop transfer function to be

$$\frac{w_c}{s + w_c}, \frac{w_c^2}{(s + w_c)^2}, \frac{w_c^3}{(s + w_c)^3}, \dots$$

w_c - Parameterization Examples

$G_p(s)$	$\frac{1}{s+1}$	$\frac{1}{s}$	$\frac{1}{s^2 + 2\mathbf{x}s + 1}$	$\frac{1}{s(s+1)}$	$\frac{1}{s^2}$
$G_c(s, \omega_c)$	$\frac{w_c(s+1)}{s}$	w_c	$w_c^2 \frac{s^2 + 2\mathbf{x}s + 1}{s(s+2w_c)}$	$\frac{w_c^2(s+1)}{s+2w_c}$	$\frac{w_c^2 s}{s+2w_c}$

Parameterization of Loop Shaping Controller



$$L(s) = G_p(s)G_c(s) = \left(\frac{s + \omega_1}{s}\right)^m \frac{1}{\frac{s}{\omega_c} + 1} \frac{1}{\left(\frac{s}{\omega_2} + 1\right)^n}$$

$$G_c(s) = \left(\frac{s + \omega_1}{s}\right)^m \frac{1}{\frac{s}{\omega_c} + 1} \frac{1}{\left(\frac{s}{\omega_2} + 1\right)^n} G_p^{-1}(s)$$

Assumptions

- The plant is minimum phase;
- $G_p(s)$ is given;
- $G_c(s)$ is ω_c -parameterized
- A transient profile is defined;
- A simulator is available

Optimization Procedure

1. Find plant frequency and gain scales, ω_p and k ;
2. Select controller $G_c(s, \omega_c)$
3. Scale $G_c(s, \omega_c)$ to obtain $G_c(s/\omega_p, \omega_c)/k$
5. Digitization and implementation in simulator;
6. Set an initial value of ω_c
7. Gradually increase ω_c until
 - a. Control signal becomes too noisy and/or too uneven
 - b. Indication of instability (oscillatory behavior)

Example

$$\ddot{y} = (-1.41\dot{y} + 23.2T_d) + 23.2u$$

$$G_p(s) = \frac{k}{\frac{s}{\mathbf{w}_p} \left(\frac{s}{\mathbf{w}_p} + 1 \right)}, \quad k=11.67, \quad \mathbf{w}_p = 1.41$$

$$\bar{G}_p(s) = \frac{1}{s(s+1)}$$

$$G_{cl}(s) = \frac{\mathbf{w}_c^2}{(s + \mathbf{w}_c)^2}$$

Control Law

$$u = k_p (r - y) + k_d (-\dot{y})$$

$$k_p = \mathbf{w}_c^2 \text{ and } k_d = 2\mathbf{w}_c - 1$$

$$k_p = \frac{\mathbf{w}_c^2}{k} = .086\mathbf{w}_c^2 \text{ and } k_d = \frac{2\mathbf{w}_c - 1}{k\mathbf{w}_p} = .061(2\mathbf{w}_c - 1)$$

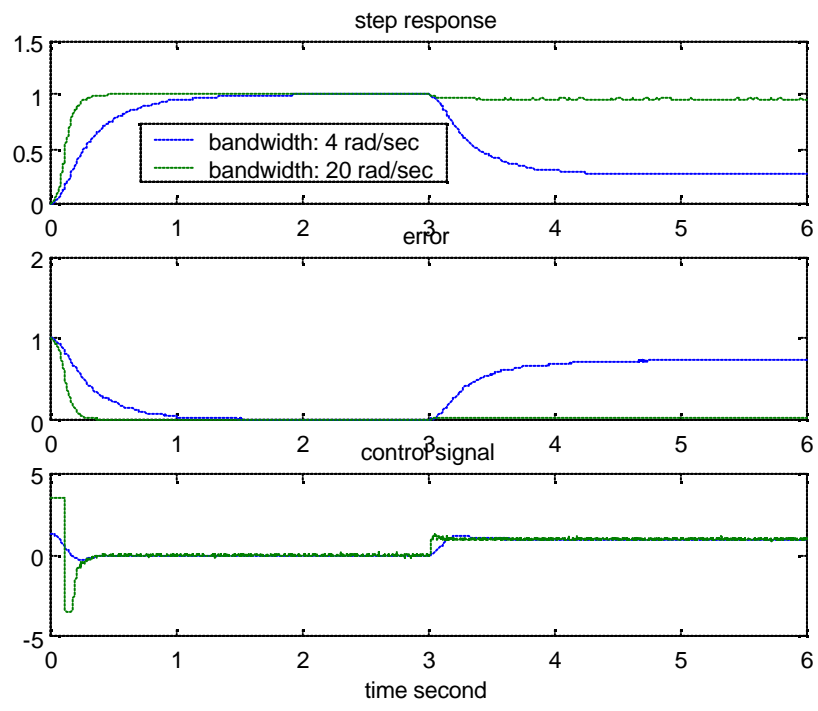
approximate differentiator: $\frac{s}{\left(\frac{s}{10\mathbf{w}_c} + 1\right)^2}$

Specs

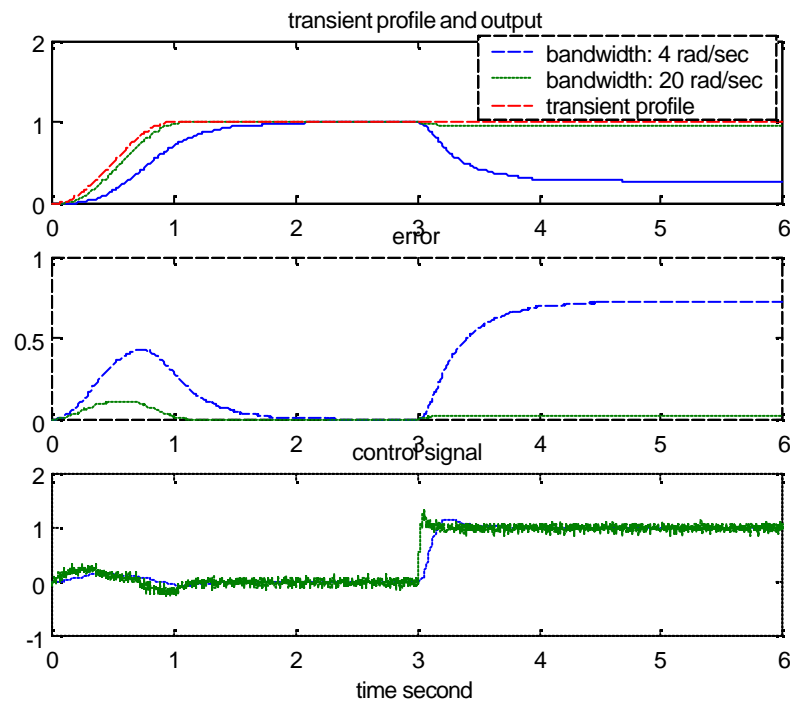
- Settling time: 1 sec
- Noise is control signal (u) not exceeding ± 100 mV
- Torque disturbance: 30%, step
- Sensor noise: .1% white noise

Simulation

Step Response



With Transient Profile



Optimization of Observer Based Design

- Optimization of the observer
- Optimization of the control law
- Combined optimization

A Generalized Disturbance Observer

$$y^{(n)} = f(t, y, \dot{y}, \dots, y^{(n-1)}, u, \dot{u}, \dots, u^{(n-1)}, w) + bu$$

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \\ \dot{x}_n = x_{n+1} + b_0 u \\ \dot{x}_{n+1} = h \\ y = x_1 \end{array} \right. \quad \left\{ \begin{array}{l} \dot{z}_1 = z_2 - \mathbf{b}_1(z_1 - y(t)) \\ \dot{z}_2 = z_3 - \mathbf{b}_2(z_1 - y(t)) \\ \dots \\ \dot{z}_n = z_{n+1} - \mathbf{b}_n(z_1 - y(t)) + b_0 u \\ \dot{z}_{n+1} = -\mathbf{b}_{n+1}(z_1 - y(t)) \end{array} \right.$$

$$z_1(t) \rightarrow y(t), z_2(t) \rightarrow \dot{y}(t), \dots, z_n(t) \rightarrow y^{(n-1)}(t)$$

$$z_{n+1}(t) \rightarrow f(t, y, \dot{y}, \dots, y^{(n-1)}, u, \dot{u}, \dots, u^{(n-1)}, w)$$

w_0 -optimization

- Observer bandwidth: ω_0

$$s^n + \mathbf{b}_1 s^{n-1} + \dots + \mathbf{b}_{n-1} s + \mathbf{b}_n = (s + \mathbf{w}_o)^n$$

- Optimization:

Maximize ω_0 until noise reaches threshold

Control Law

$$u = -\frac{z_{n+1} + u_0}{b_0}$$

$$y^{(n)} = (f - z_{n+1}) + u_0 \approx u_0$$

$$u_0 = k_p (r - z_1) - k_{d_1} z_2 - \dots - k_{d_{n-1}} z_n$$

$$s^n + k_{d_{n-1}} s^{n-1} + \dots + k_{d_1} s + k_p = (s + \mathbf{w}_c)^n$$

Design Procedure

- Step 1: Design a parameterized observer and feedback controller;
- Step 2: Design a transient profile with the equivalent bandwidth of ω_{ct} ;
- Step 3: Select an ω_0 to 5-10 times larger than ω_{ct} ;
- Step 4: Set $\omega_c = \omega_0$ and increase both by the same amount until the noises levels and/or oscillations in the control signal and output exceed the tolerance;
- Step 5: Incrementally increase or decrease ω_c and ω_0 individually, but keep the sum the same.

Example

$$\ddot{y} = (-1.41\dot{y} + 23.2T_d) + (23.2 - 40)u + 40u = f + 40u$$

$$\dot{z} = \begin{bmatrix} -3\mathbf{w}_o & 1 & 0 \\ -3\mathbf{w}_o^2 & 0 & 1 \\ -\mathbf{w}_o^3 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 3\mathbf{w}_o \\ 40 & 3\mathbf{w}_o^2 \\ 0 & \mathbf{w}_o^3 \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix}$$

$z_1 \rightarrow y, z_2 \rightarrow \dot{y},$ and

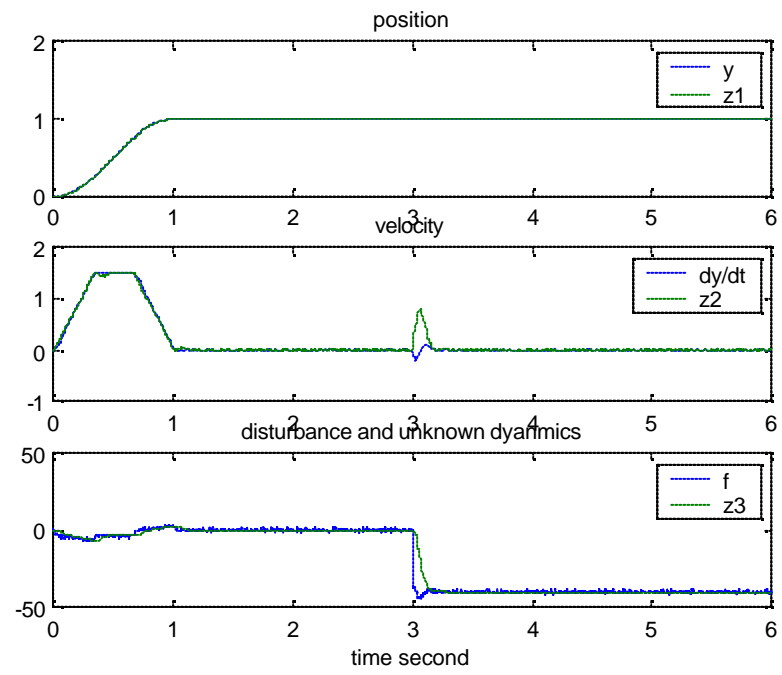
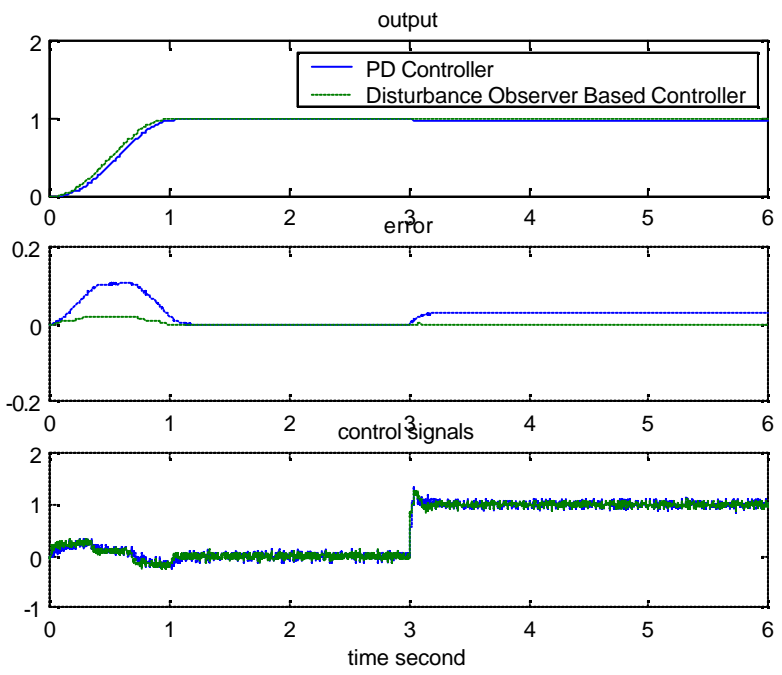
$z_3 \rightarrow f = -1.41\dot{y} + 23.2T_d + (23.2-40)u,$ as $t \rightarrow \infty$

$$u = \frac{u_0 - z_3}{40}$$

$$u_0 = k_p (r - z_1) - k_d z_2$$

$$k_d = 2\mathbf{x}\mathbf{w}_c, \quad \mathbf{x} = 1, \quad \text{and} \quad k_p = \mathbf{w}_c^2$$

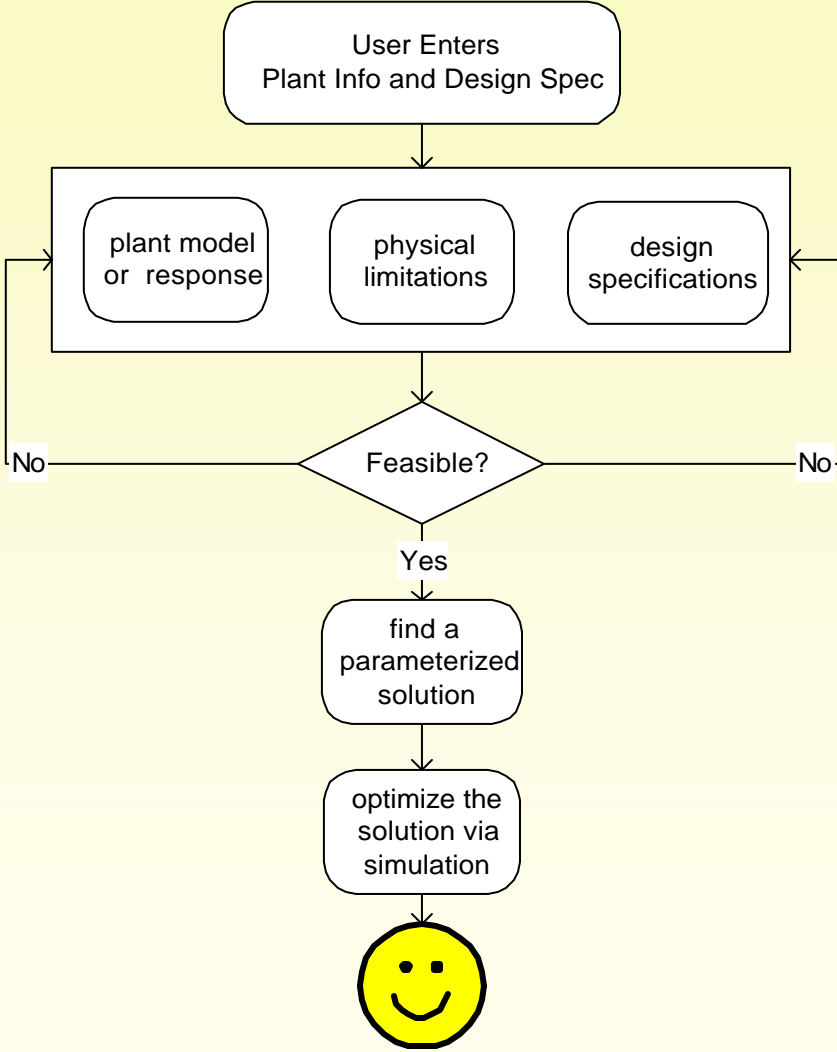
Simulation



Self-Tuning

- ω_0
Automatically response to the noise level
- ω_c
Bandwidth scheduling
- Scaling Factors (k, ω_p)
Adaptation to plant dynamics variations

CAD Package Blueprint



Summary

- Three new concepts
 - Scaling
 - Parameterization
 - Practical Optimization
- Applications
 - From art to science
 - Stops reinventing the wheels
 - Optimization in real world