Linear Active Disturbance Rejection Controller (LADRC)

Plant: $\ddot{y} = -a\dot{y} - by + w + bu$

$$\ddot{y} = -a\dot{y} - by + w + (b - b_0)u + b_0 u$$

$$= f + b_0 u, \quad f = -a\dot{y} - by + w + (b - b_0)u$$

State Space

$$x_1 = y,\ x_2 = \dot{y},\ x_3 = f,\ h = \dot{f}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + b_0 u \\ \dot{x}_3 = h \\ y = x_1 \end{cases} \Rightarrow \begin{array}{l} \dot{x} = Ax + Bu + Eh \\ y = Cx \end{array}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b_0 \\ 0 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0], \quad E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Linear Extended State Observer (LESO)

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\hat{y} = C\hat{x}$$

$$L = [\beta_1 \;\; \beta_2 \;\; \beta_3]: \text{ observer gain}$$

$$\hat{x}: \text{ estimated state}$$

$$\hat{x} \to x$$

Control Law

$$u = (u_0 - \hat{x}_3)/b_0, \quad \hat{x}_3 \to f$$

$$\Downarrow$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u_0 \\ y = x_1 \end{cases}$$

$$u_0 = k_p(r - \hat{x}_1) - k_d \hat{x}_2$$

Parameterization of ESO

- Observer bandwidth: ω_o

$$\lambda_o(s) = s^3 + \beta_1 s^2 + \beta_2 s + \beta_3 = (s + \omega_o)^n$$

- $\beta_1 = 3\omega_o,\ \ \beta_2 = 3\omega_o^2,\ \ \beta_3 = \omega_o^3$

Discrete ESO

$$\dot{x} = Ax + Bu + Eh$$
$$y = Cx$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b_0 \\ 0 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0], \quad E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$
$$y(k) = Hx(k) + Ju(k)$$

$$\Phi = e^{AT} \quad \Rightarrow \quad \sum_{k=0}^{\infty} \frac{A^k T^k}{(k)!}$$

$$\Gamma = \int_0^T e^{A\tau} d\tau \, B \quad \Rightarrow \quad \sum_{k=0}^{\infty} \frac{A^k T^{k+1}}{(k+1)!} B$$

$$H = C, \quad J = 0$$

$$\Phi = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} bT \\ b \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Discretization of the Plant

$$\dot{x} = Ax + Bu + Eh$$
$$y = Cx$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b_0 \\ 0 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0], \quad E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Discrete ESO

Plant

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$
$$y(k) = Hx(k) + Ju(k)$$

DESO

$$\hat{x}(k+1) = \Phi \hat{x}(k) + \Gamma u(k) + L_p e(k)$$
$$\hat{y}(k) = H\hat{x}(k) + Ju(k)$$

CDESO

$$\bar{x}(k) = \hat{x}(k) + L_c e(k)$$
$$L_p = \Phi L_c$$
$$\hat{x}(k+1) = \Phi \bar{x}(k) + \Gamma u(k)$$

$$\lambda(z) = \left| zI - (\Phi - \Phi L_c H) \right| = (z-\beta)^{n+h}$$
$$L_c = \left[1-\beta^3,\ (1-\beta)^2(3+3\beta)\tfrac{1}{2T},\ (1-\beta)^3\tfrac{1}{T^2} \right]^T$$
$$\beta = e^{-\omega_o T}$$