Application of Classical Controller Design Techniques to an Industrial Positioning System

***Abstract*—A positioning control system for an industrial application is designed and simulated for the use with a PID and loop-shaping controller. Classical design techniques are applied and their control performance under various conditions is assessed. The simulation is carried out under different input and disturbance conditions, as well as resonance mode additions.**

Keywords: PID controller; loop shaping, frequency response

I. INTRODUCTION

The methods of classical controller design, such as the celebrated frequency response method, have a well-established theoretical background. But their application in industry has been very sparse, save for the Proportional-Integral-Derivative (PID) controller which accounts for over 90% of all industrial controller applications. Here we use a real-world industrial positioning system to make a comparison between PID and loop-shaping controller design. Our approach is problem-oriented, with the main focus on system performance, not the demonstration of what a particular control design is capable of. We want to compare controller performance under various inputs and disturbances, as well as the amount of tuning effort spent in obtaining the desired performance under the different conditions. This will give us a good idea of why the PID controller is so widely used in industry and a glimpse of how to excel it.

II. PLANT MODELING

The plant (figure 1) is based on a belt-and-pulley mechanism, driven by a DC motor. It is to move a load of 235lbs by 12 inches in 0.3 seconds with no overshoot. Plant and control saturations have been included in the model.

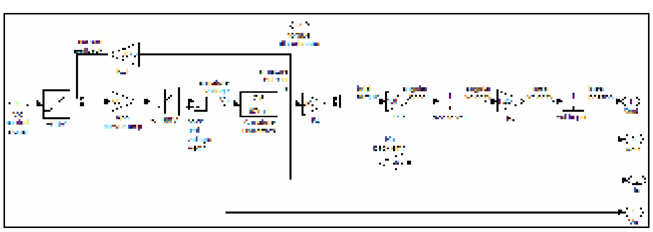

Figure 1. SIMULINK Model of Plant

From the plant model, a closed-loop PID control is constructed (figure 2). All the components in the plant simulation, including the parameters and saturations have been provided by Gao [1]. The closed-loop system design has been modified after Gao's design. The modifications allow for different inputs and disturbances through simple manual switches.

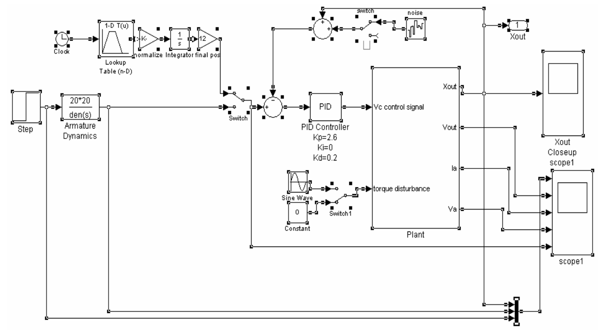

Figure 2. SIMULINK Model of Closed-loop Control

III. PID TUNING

The PID controller was proposed in 1922 by N. Minorsky [2] based on intuitive arguments of error feedback. Its control law is given by (1).

$$u = K_p e + K_i \int e + K_d \dot{e} \tag{1}$$

The tuning of PID is generally based on a set of empirical rules of thumb. Generally speaking the proportional gain K_p is first tuned to give an adequate response speed, while the differential gain K_d is then used to decrease the overshoot. Tuning the integral gain K_i may remove steady state error. The PID is retuned depending on the input profiles used.

A. Step input

For the initial simulation with a step input of 12V (at t=0) and no external disturbance and noise, different PID parameter sets with similar performance results were compared. Several of them do not require the integral part, which hints at the fact that the plant already contains one integrator for position. It is decided that the set with $K_p = 2.2, K_i = 0, K_d = 0.168$ be adopted, for the benefit of small gain values. The voltage and current response are comparatively smoother than other parameter sets. The position response clearly satisfies the settling time of 0.3s (figure 3).

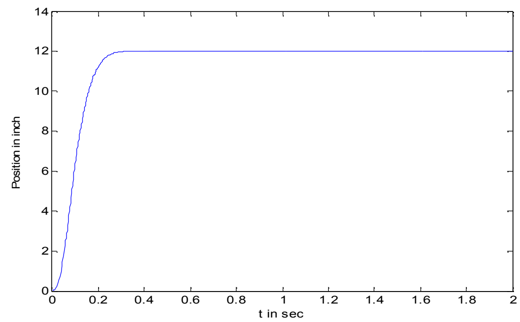

Figure 3. Position Response under Step Input

In order to achieve settling times less than 0.3s, the proportional and differential gains were tuned up. For instance the parameter set $K_p = 2.6, K_i = 0, K_d = 0.2$ gives a better settling time. The attempt to lower it below approximately 0.2s was however unsuccessful. The observed control voltage has an initial spike reaching more than 5000V, which increases with increasing controller gains.

There is always a trade-off between settling time performance and the armature voltage and current. The position response can be improved at a cost of large unsteadiness in current and voltage.

B. *Addition of a low pass prefilter.*

In order to decrease control and armature voltages, a low pass prefilter with unity gain is inserted directly after the step input. It is chosen to be a second order filter, such that the damping factor is unity (to achieve critical damping). The natural frequency which gives rise to a settling time of approx. 0.3 seconds is empirically determined to be $\omega = 26$ rad/s, in discordance with the theoretical value of 13.33 rad/s. This is presumably due to the fact that the second order filter has a dominant pole which renders the settling time formula inaccurate. In order to track the prefilter's motion profile, the PID has to be retuned to $K_p = 0.8, K_i = 0, K_d = 0.16$ (for results see figure 4). There is significant change in the motor voltage and current amplitude, the initial spikes become smaller. The control voltage decreased by an entire magnitude.

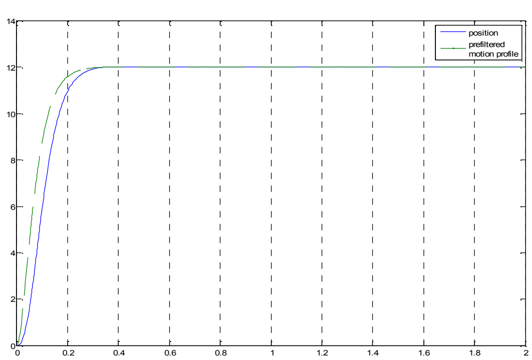

Figure 4. Prefiltered motion profile and position tracking

C. *Trapezoidal velocity profile.*

When the low-pass filter is replaced by a standard trapezoidal velocity profile, the armature voltage and current are much smoother and smaller than either of the previously used profiles (figure 5). The resulting control voltage stays within +5V and –5V. The same smoothness is evident in the velocity output. This fact makes this profile widely used in the industry, since it minimizes deleterious abrupt changes in system dynamics. The trapezoidal profile is defined in Simulink by the 1-D look-up table with the input vector [0 0.1 0.2 0.3 0.4] and the output vector [0 1.5 1.5 0 0]. The PID has to be slightly retuned for the position output to follow this profile without overshoot.

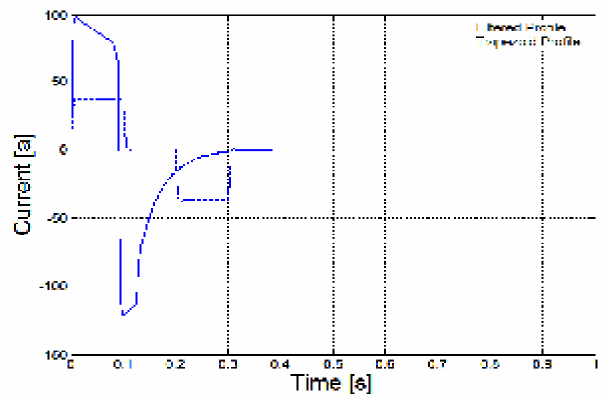

Figure 5. Effect of Motion profiles on Armature Current

The comparison of various motion profiles illustrates the strong effect they may have on the dynamics of the system. The choice of motion profile will thus affect the controller performance. This results in the need to retune the PID gains for each of the three motion profiles above to achieve optimal performance. A fair comparison of different controllers must therefore be based on the same motion profiles.

IV. Controller Performance under Disturbance

Control systems are always subject to external disturbance and internal noise which affects the dynamics. If the nature of the disturbance is known, they can be modeled mathematically. In the positioning system, this is done by adding torque disturbance to the motor and sensor noise to the position sensor. In practice however, the nature of most disturbances is not known and may not be easily modeled in simulation. The performance of the PID controller is assessed through simulation under various known forms of disturbance and noise. To have a consistent ground for comparison, the prefiltered step motion profile from Section III.B is used.

A. Sensor noise.

A small, random noise is introduced in the simulation, whose purpose is to mimic sensor noise. In Simulink, this is achieved through adding a "Band-limited White Noise" block into the feedback branch from the position to the controller, i.e., where the position sensor is normally located. The prefilter and the corresponding PID tuning is used. When a noise of 0.1% of the final position values is added (i.e. a value of 0.012 inch is set in the Simulink noise box), the response still maintained its stability, while deviating within 2% from its steady state value of 12 inches. However when the sensor noise is gradually increased to 10%, the response looses integrity, and deviations to almost 16 inches are seen (figure 6). The current and voltage response become unsteady even under 0.1% noise, exhibiting sharp peaks.

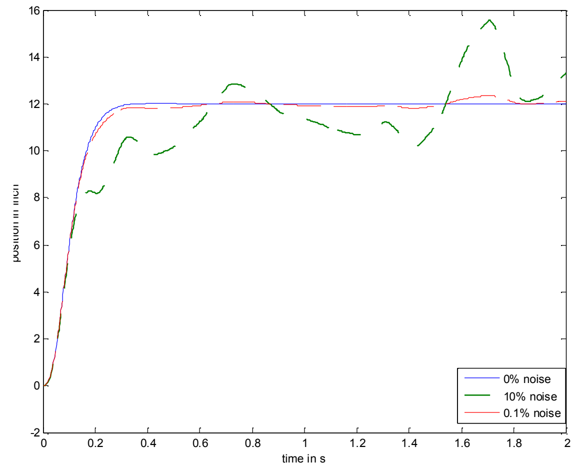

Figure 6. Effects of various degrees of sensor noise

B. Constant and Sinusoidal Torque Disturbances.

A *constant* torque disturbance is added into the plant at the total torque summing junction (see figure 1). Its onset is at time t = 0. The value of the torque disturbance, given in English units, is $lb * inch / \sec$. The steady state error grows with the disturbance. Above a value of 1000, the position response grows out of bound (figure 6). This is especially striking at a value greater than 1500. The current and voltage response are manifested through a hefty initial rise.

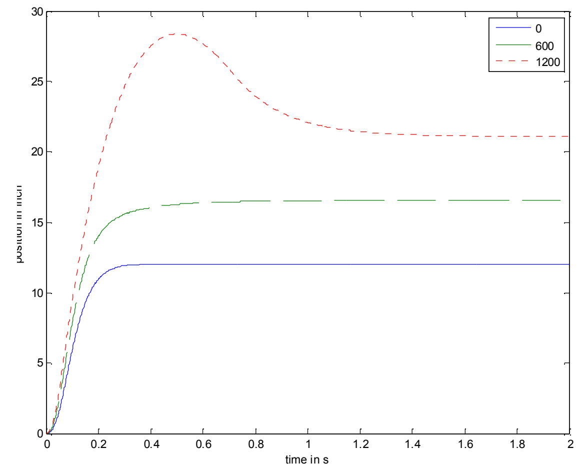

Figure 7. Effects of various degrees of constant disturbance

The constant torque is replaced by a *sinusoidal* torque with an amplitude of 10% of the maximum motor torque. The motor torque is related to the maximum current via the relation $\tau = K_t I_a$. The value of the torque constant K_t is 13.2 $lb * inch / A$. The maximum measured current of nearly 100A indicates a maximum torque disturbance of 1320 $lb * inch / \sec$. The lower the disturbance frequency, the more drastic is the effect on the position response. While any disturbance above 10Hz becomes insignificant, resulting in only < 2% steady state variation, frequencies below 5Hz make the response increasingly unstable, with 1Hz being the lower limit (figure 8). An interesting synopsis is obtained by comparing the frequency variation at the amplitude of 1320 to constant torque disturbance of the same amplitude: the constant disturbance reaches a maximal initial position deviation up to 25 inches, whereas the sinusoidal disturbance achieves the same deviation only at a frequency of 1Hz or lower.

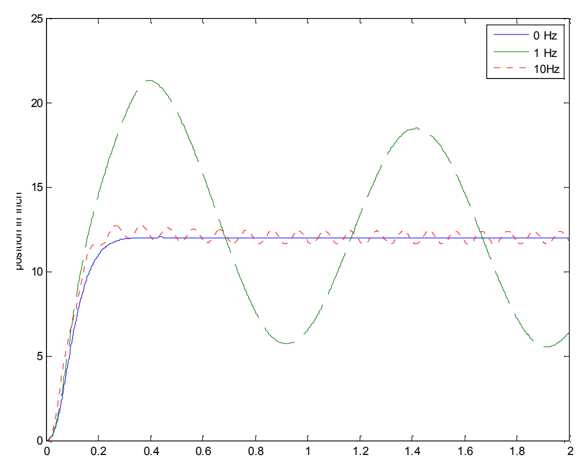

Figure 8. Effects of various degrees of sinusoidal disturbance

V. LOOP-SHAPING CONTROLLER DESIGN

Among the classical controller design methods, Loop-Shaping is the only existing analytical technique that can address most of design specifications and constraints at once. The term loop-shaping [4], [5] refers to the manipulation of the open-loop gain frequency response (2), as a design tool.

$$L(s) = C(s)G(s) \tag{2}$$

Given the transfer function of the plant $G(s)$, the goal is to design a controller $C(s)$, such that the open-loop gain $L(s)$ attains the ideal Bode Gain Plot given in Figure 9. Such a controller is also termed a *compensator*. The design specifications and constraints are addressed as follows. At low frequency, the loop gain should be big, to achieve low steady state error; whereas in the high frequency domain, a low gain is required to attenuate noise. In the middle section where the bandwidth of the system lies, the slope of the Bode Gain Plot should be -20dB/dec, to mimic the effect of an integrator. The speed of the plant response is reflected in the choice of an appropriate bandwidth. The stability margins of the open-loop are to be kept above a reasonable range.

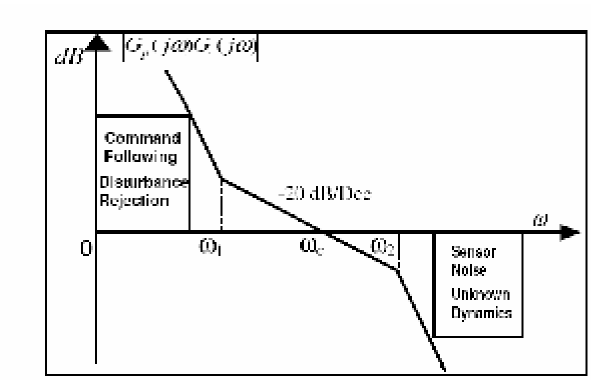

Figure 9. Ideal Bode Plot Shape

A. Characterization of PID Loop

The PID controller may be characterized through a frequency response analysis of the PID-loop gain, L(s). Based on equation 2, Bode plots are graphed for $L=CG$, where C is the PID controller, and G the positioning plant transfer function. Through simple block diagram reduction of figure 1, the transfer function for position is obtained as:

$$G_p(s) = \frac{165000}{s(s+796)(s+3.085)} \tag{3}$$

Equation 3 shows that the plant is third order, with second order dominant poles at the origin and 3.1. The PID transfer function can be obtained from (1)

$$C_{PID}(s) = K_p + K_i / s + K_d s \tag{4}$$

whereby the PID gains are {0.8, 0, 0.16}. Multiplying (3) and (4) gives the PID-loop gain.

$$L_{PID} = \frac{26400s + 82500}{s(s+796)(s+3.085)} \tag{5}$$

From equation (5), the bode plot, and hence the gain and phase margins can be determined (figure 10). The infinite gain margin and a phase margin of 87.6 degrees bespeak of the robust stability of this PID loop.

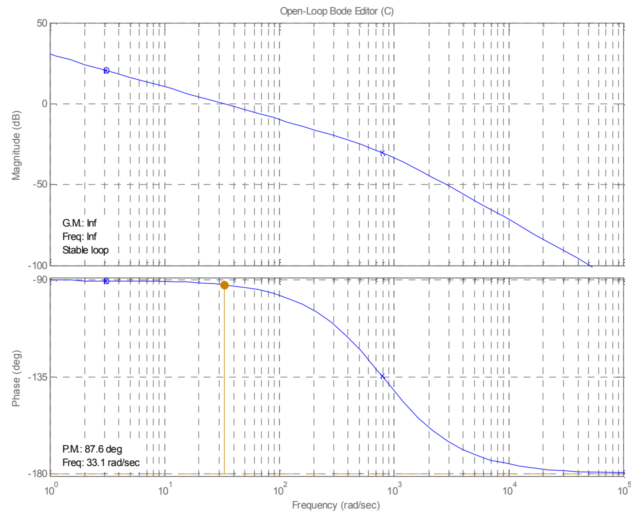

Figure 10. Bode Plots of PID -Loop

The loop bandwidth of approximately 33.1 rad/s points toward a settling time of $t_s \approx 4/\omega_n = 0.12\,\text{sec}$. The fact that the actual position response simulated in Part I was close to 0.3sec may be due to the existing saturations in the plant. It should be reminded that the frequency response only applies to linear systems. Thus the loop-shaping method cannot adequately address non-linearities in plant dynamics such saturation.

The bandwidth of the PID loop gives us an idea of how to improve the controller using loop shaping design: The new open-loop transfer function should have a bandwidth significantly greater than 33.1 rad/s, while still maintaining proper stability margins. In particular, a minimum phase margin of 60 deg is recommended.

B. Compensator Design

The design of the compensator requires extensive manipulation of Bode plots. An advanced graphical user interface tool, Matlab SISOTool, has been employed. The resulting compensator (6) and Bode plots (figure 11) show very stable gain margin of 28dB and phase margin of 73 degrees.

$$C_{Comp} = \frac{0.19s + 1}{0.005s + 1} \tag{6}$$

The approximate bandwidth is 39.4 rad/sec, which corresponds theoretically to a settling time of 0.1s. However, simulations employing the three motion profiles from Part I all resulted in a settling time equal or greater than 0.3s.

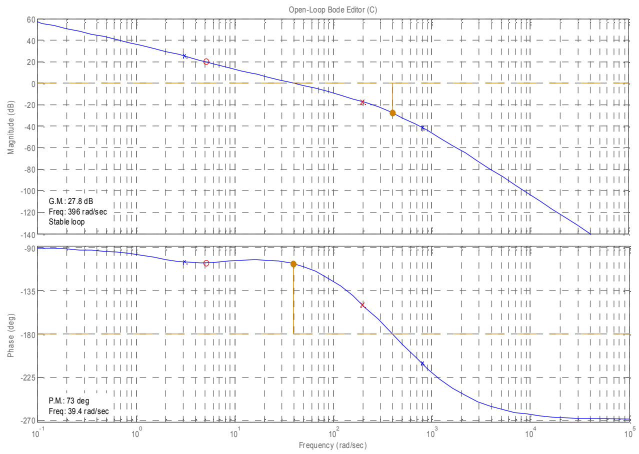

Figure 11. Bode Plots of Compensator-Loop

As in the case with the PID, the discrepancy between theoretical and actual performance can be traced back to plant saturations. The range of control voltage saturation is +-8V, beyond which the plant becomes highly non-linear. The initial control signal following step input reaches more than 200V. However, when the saturations were removed from the plant model, the compensator works perfectly and achieves a settling time below 0.1s under a step input.

To achieve the control design specifications while keeping the plant saturations, a trapezoidal velocity profile is used. The trapezoid employed in Section III.C with the input vector [0 0.1 0.2 0.3 0.4] was designed to give a settling time of 0.3s. The control and armature voltage it creates are by far the lowest among all three motion profiles examined. Choosing the input vector to be [0 0.05 0.1 0.15 0.4] speeds up the intended settling time to below 0.2s. The control voltage moved within the range of -15V and +15V, only slightly above the saturation limit.

C. Comparing PID and Loop-Shaping Controller

Although very desirable, a direct, quantitative comparison between the PID and loop-shaping controller performance is very difficult to implement. This is due to the fact that PID and loop-shaping controller lack optimal tuning strategies. Without the knowledge of the controller parameter set which leads to optimal performance, a direct comparison between the controller performances cannot be made. Complicating matters even further, the loop-shaping technique does not allow straightforward tuning in the sense of tuning the gains of the PID. Rather it relies on the often tedious process of finding the adequate poles and zeros, entirely based on graphical manipulation of Bode plots.

To address these issues, Zheng and Gao [6] have devised a technique to optimize controller parameters using Genetic Algorithm. A quantitative comparison based on cost function minimization leads to the conclusion that a parameterized version of the loop-shaping technique [7] outperforms the PID controller. Judged from the cost function, the loop shaping technique is 23% more efficient than the classical PID control. However, such a comparison does not include the associated manual tuning effort for finding the optimal parameters.

VI. Plant with Resonant Mode

The phenomenon of resonance leads to a sharp increase in system excitation and associated instability. It happens when a periodic input of a certain frequency causes a sudden rise in plant gain. The frequency at which the plant becomes vulnerable to such inputs is called the natural frequency. In this section we want to characterize the loop-shaping technique in terms of its ability to address resonance. For the simulation, the transfer function block (7) is inserted into the plant model, between torque and acceleration. Such a block is referred to as a resonant mode.

$$\frac{\omega_n^{\,2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\,2}} \qquad (7)$$

With $\zeta = 0.05$ and with a natural frequency of $\omega_n = 50$ and $\omega_n = 100\pi$. The addition modifies the plant transfer function to

$$G_{50} = \frac{412500000}{s^5 + 800s^4 + 6500s^3 + 2000000s^2 + 6100000s} \qquad (8)$$

$$G_{100\pi} = \frac{16000000000}{s^5 + 834s^4 + 124000s^3 + 80000000s^2 + 242000000s} \qquad (9)$$

for $\omega_n = 50$ and $\omega_n = 100\pi$ respectively. If the modes are multiplied together, then the plant will exhibit resonances at both frequencies. For the sake of illustrative purposes, we address the modes separately.

The Bode amplitude plots show peak at the natural frequency while the phase plots show sudden vertical dip over 180 degrees (Figure 12).

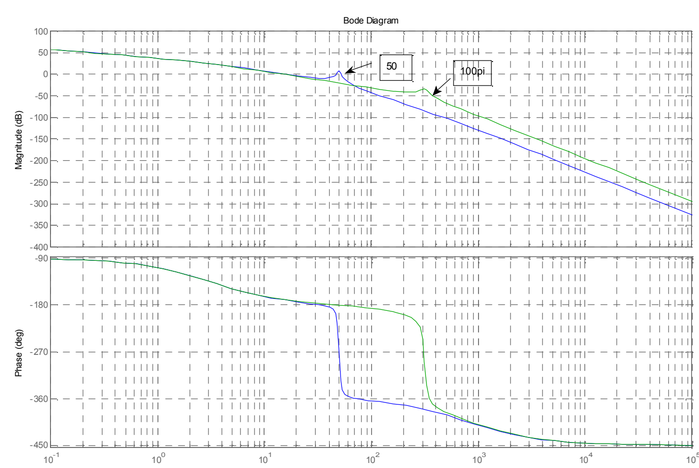

Figure 12. Bode Plots of Plant with resonant modes

The presence of resonant modes makes simple poles and zeros less effective in loop-shaping. The adopted strategy is to move the resonant mode as far away from the 13rad/sec bandwidth as possible (see figure 13), while still maintaining reasonable stability margins. This was achieved for the resonance occurring at $\omega_n = 100\pi$, with a compensator given by (10). The crossover frequency is 24.6 rad/sec. In the simulation a settling time very close to 0.2 seconds was achieved under a step input.

$$C_R = \frac{0.12*(0.086s+1)}{(0.0031s+1)(0.003s+1)} \qquad (10)$$

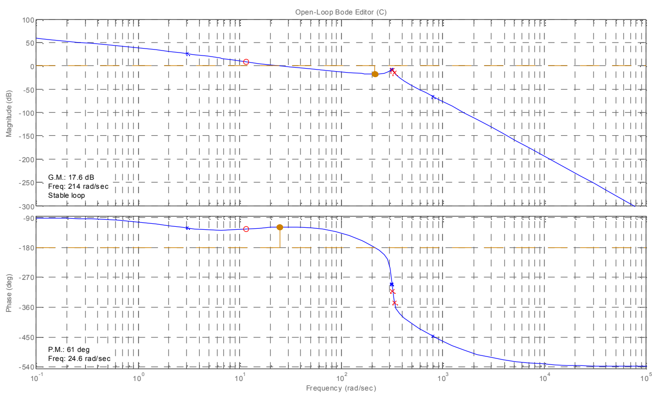

Figure 12. Bode Plots of Plant with resonant modes

It proved to be unfeasible to move the resonant mode at $\omega = 50$ away from the bandwidth of 13rad/s, while still maintaining reasonable stability margins. The resulting simulations always exhibited drastic position overshoot. This fact exemplifies the following rule concerning loop-shaping design:

"bandwidth is severely constricted by the location of the resonant mode".

VII. Conclusion and Future Work

PID and loop-shaping controller performances have been compared through application to a realistic industrial positioning plant model, with specific design criteria. The model is simulated in Matlab Simulink under various input and disturbance conditions. The performance of both control methods are contingent on the particular input motion profiles and external disturbances. In the case of the PID, the controller gains may be easily readjusted to accommodate each condition, though the tuning is based entirely on trial and error. The loop-shaping control design, which has its basis in frequency response analysis, is based on manipulating the Bode gain plot of the compensator loop. In theory, this mathematically motivated method successfully addresses most of the design criteria, even the existence of certain resonant modes. Yet its actual performance is drastically hampered by plant saturations and other non-linearities in dynamics. It cannot be tuned in the sense of tuning a PID, which limits its flexibility and ease of use under a change of input profile. However, it may be shown through advanced mathematical optimization techniques that a certain class of loop-shaping controller significantly outperforms PID controllers.

Our analysis of these issues leads to the conclusion that there is a chasmic trade-off between performance and ease-of-use. The industrial success of the PID controller may be largely due to its flexibility in tuning which gives it a very wide range of applications. In contrast, the loop-shaping controller is very plant-specific. However, neither one addresses the problem of uncertainties in plant dynamics well.

These facts render us with a hint of what a successful control strategy should feature:

a) the control technique must be versatile, i.e., it cannot be plant-specific.

b) the controller must be easy to use, with a small number of tuning parameters

c) the control technique should be capable of addressing change in plant dynamics

As daunting as these tasks may appear, for all we know, a solution may be already in store [6].

VIII. References

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