V. Practical Optimization

- Scaling
- Practical Optimality
- Parameterization
- Optimization
  - Objectives
  - Means
  - Solutions
    - ω_c-optimization
    - Observer based design

Definition of Gain and Frequency Scales

$$G_p(s) \Rightarrow kG_p(s)$$

$$G_p(s) \Rightarrow G_p(s/\omega_p)$$

$$\frac{23.2}{s(s+1.41)} = \frac{11.67}{\frac{s}{1.41}(\frac{s}{1.41}+1)} = \frac{k}{\frac{s}{\omega_p}(\frac{s}{\omega_p}+1)}$$

$$k = 11.67$$

$$\omega_p = 1.41$$

Unit-Gain Unit-Bandwidth Plants

$$\frac{1}{s+1}, \frac{1}{s}, \frac{1}{s^2+2\xi s+1}, \frac{1}{\mathrm{s(s+1)}}, \frac{1}{\mathrm{s}^2}, \frac{1}{s^3+\xi_1 s^2+\xi_2 s+1}, \ \cdots$$

$$\frac{s+1}{s^2+2\xi s+1}, \frac{s^2+2\xi_z s+1}{s^3+\xi_1 s^2+\xi_2 s+1}, \ \cdots$$

- Controllers for UGUB plants are predetermined

Scaling of a controller for a new plant

- Stop reinventing the wheel
- A class of processes/plants, differ only in dc-gain and bandwidth
- Normalization of plants: unit-gain unit-bandwidth
- Controller design for UGUB plants
- Controller is obtained by scaling UGUB controllers

$$\{G_p(s), G_c(s)\} \Rightarrow \{kG_p(s/\omega_p), G_c(s/\omega_p)/k\}$$

Scaling of PID

$$\{G_p(s), G_c(s) = k_p + \frac{k_i}{s} + k_d s\}$$

$$\Downarrow$$

$$\{kG_p(s/\omega_p), G_c(s) = (k_p + k_i \frac{\omega_p}{s} + k_d \frac{s}{\omega_p})/k\}$$

$$\Downarrow$$

$$\overline{k_p} = \frac{k_p}{k}, \overline{k_i} = \frac{k_i \omega_p}{k}, \overline{k_d} = \frac{k_d}{k\omega_p}$$

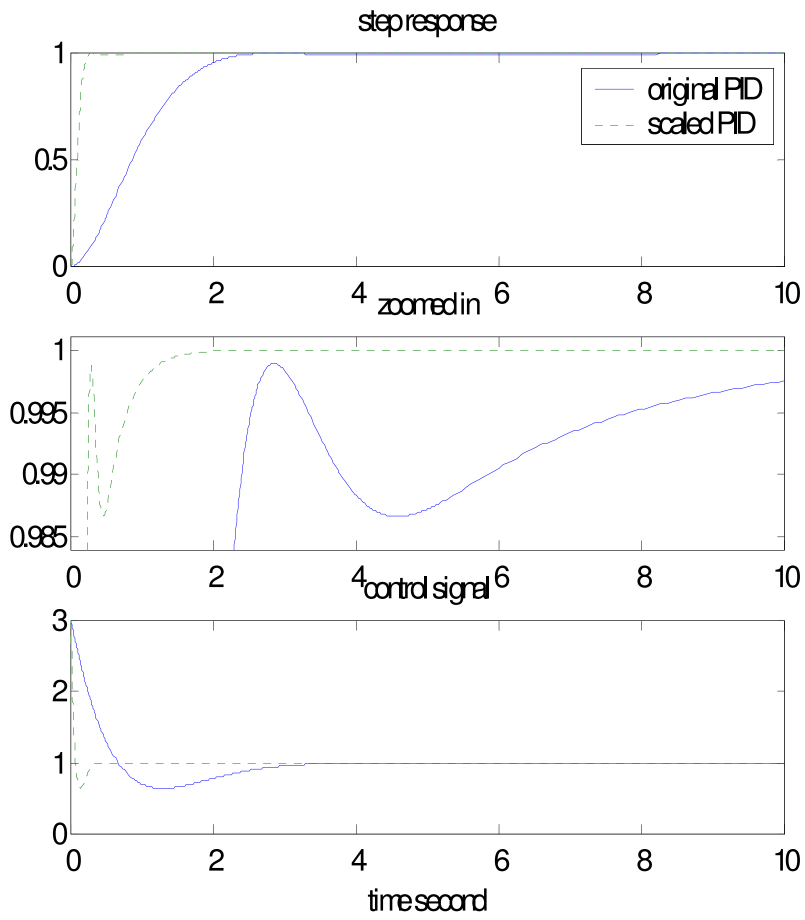
Scaling Example

$$G_p(s) = \frac{1}{s^2 + s + 1}$$

k_p=3, k_i=1, and k_d = 2.

$$G_p(s) = \frac{1}{(\frac{s}{10})^2 + \frac{s}{10} + 1}$$

$$\overline{k}_p = 3, \overline{k}_i = 10, \overline{k}_d = .2$$



Practical Optimality

Optimization of Control Law:

To maximizing a function of given performance measures subject to the physical limitations of the design

Quality of Control:

Speed, Accuracy,

Disturbance Rejection

Constraints:

Sampling Rate, Sensor Noise, Uncertainty, Actuation Smoothness, Saturations

ω_c-Optimization

maximizing the loop gain bandwidth, ω_c, subject to the physical limitations of the design.

ω_c- Parameterization

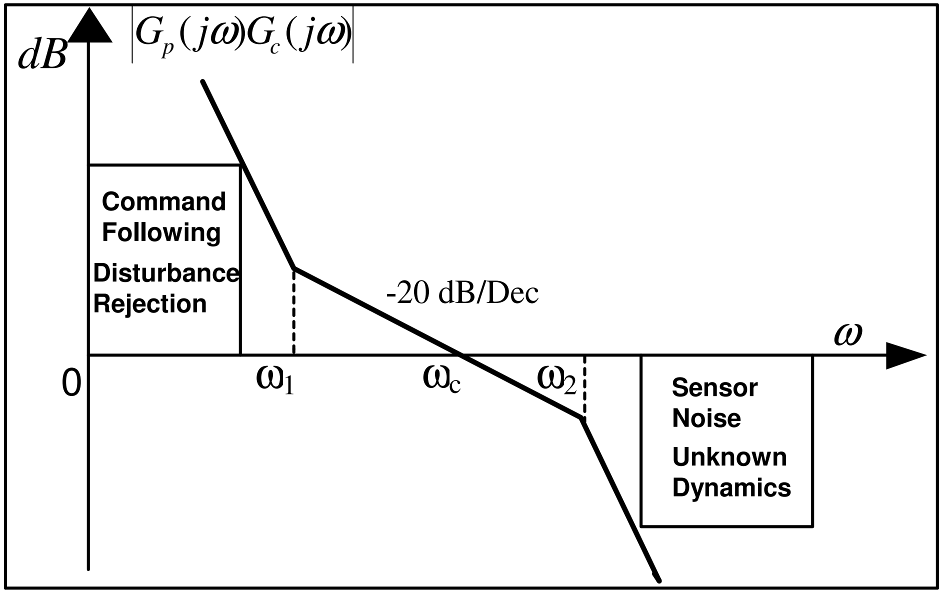
parameterization of the controllers, $G_c(s,\omega_c)$, for the unit gain and unit bandwidth that result in the loop gain bandwidth to be ω_c, or closed-loop transfer function to be

$$\frac{\omega_c}{s+\omega_c},\ \frac{\omega_c^2}{(s+\omega_c)^2},\ \frac{\omega_c^3}{(s+\omega_c)^3},\ \cdots$$

ω_c- Parameterization Examples

$G_p(s)$	$\frac{1}{s+1}$	$\frac{1}{s}$	$\frac{1}{s^2+2\xi s+1}$	$\frac{1}{s(s+1)}$	$\frac{1}{s^2}$
$G_c(s,\omega_c)$	$\frac{\omega_c(s+1)}{s}$	ω_c	$\omega_c^2\frac{s^2+2\xi s+1}{s(s+2\omega_c)}$	$\frac{\omega_c^2(s+1)}{s+2\omega_c}$	$\frac{\omega_c^2 s}{s+2\omega_c}$

Parameterization of Loop Shaping Controller



$$L(s) = G_p(s)G_c(s) = \left(\frac{s+\omega_1}{s}\right)^m \frac{1}{\dfrac{s}{\omega_c}+1} \frac{1}{\left(\dfrac{s}{\omega_2}+1\right)^n}$$

$$G_c(s) = \left(\frac{s+\omega_1}{s}\right)^m \frac{1}{\dfrac{s}{\omega_c}+1} \frac{1}{\left(\dfrac{s}{\omega_2}+1\right)^n} G_p^{-1}(s)$$

Assumptions

- The plant is minimum phase;
- $G_p(s)$ is given;
- $G_c(s)$ is ω_c-parameterized
- A transient profile is defined;
- A simulator is available

Optimization Procedure

1. Find plant frequency and gain scales, ω_p and k;
2. Select controller $G_c(s, \omega_c)$
3. Scale $G_c(s, \omega_c)$ to obtain $G_c(s/\omega_p, \omega_c)/k$
5. Digitization and implementation in simulator;
6. Set an initial value of ω_c
7. Gradually increase ω_c until
   a. Control signal becomes too noisy and/or too uneven
   b. Indication of instability (oscillatory behavior)

Example

$$\ddot{y} = (-1.41\dot{y} + 23.2T_d) + 23.2u$$

$$G_p(s) = \frac{k}{\frac{s}{\omega_p}(\frac{s}{\omega_p}+1)},\ \text{k}=11.67,\ \omega_{\text{p}} = 1.41$$

$$\bar{G}_p(s) = \frac{1}{s(s+1)}$$

$$G_{cl}(s) = \frac{\omega_c^2}{(s+\omega_c)^2}$$

Control Law

$$u = k_p(r - y) + k_d(-\dot{y})$$

$$k_p = \omega_c^2 \text{ and } k_d = 2\omega_c - 1$$

$$k_p = \frac{\omega_c^2}{k} = .086\omega_c^2 \text{ and } k_d = \frac{2\omega_c - 1}{k\omega_p} = .061(2\omega_c - 1)$$

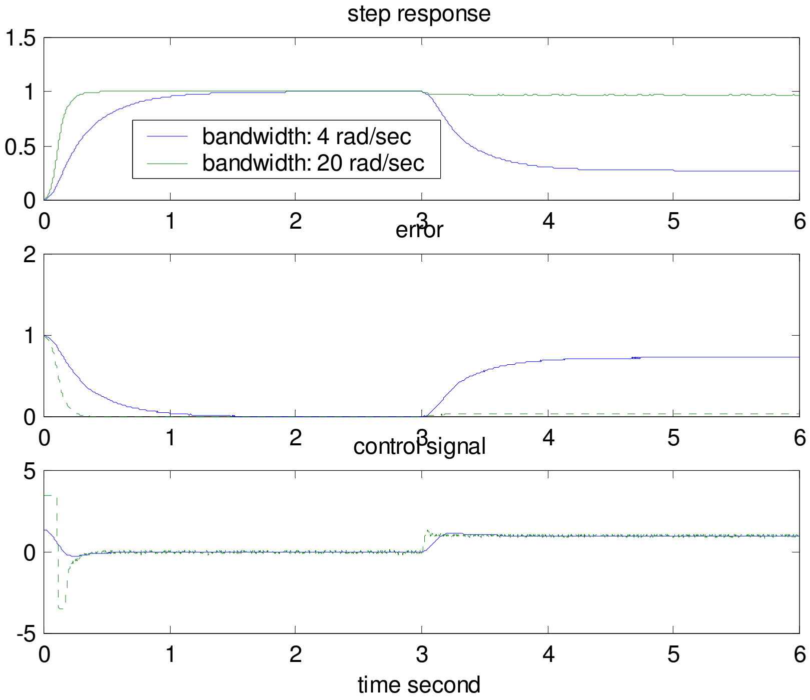
approximate differentiator: $\dfrac{s}{(\frac{s}{10\omega_c} + 1)^2}$

Specs

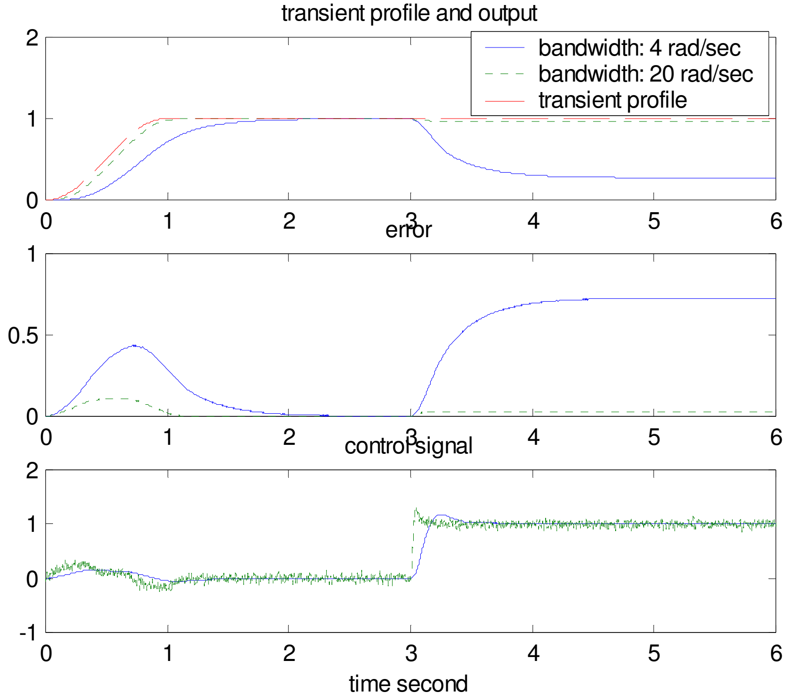
- Settling time: 1 sec
- Noise is control signal (u) not exceeding ±100 mV
- Torque disturbance: 30%, step
- Sensor noise: .1% white noise

Simulation

Step Response



With Transient Profile



Optimization of Observer Based Design

- Optimization of the observer
- Optimization of the control law
- Combined optimization

A Generalized Disturbance Observer

$$y^{(n)} = f(t, y, \dot{y}, \cdots, y^{(n-1)}, u, \dot{u}, \ldots u^{(n-1)}, w) + bu$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots \\ \dot{x}_n = x_{n+1} + b_0 u \\ \dot{x}_{n+1} = h \\ y = x_1 \end{cases}$$

$$\begin{cases} \dot{z}_1 = z_2 - \beta_1 (z_1 - y(t)) \\ \dot{z}_2 = z_3 - \beta_2 (z_1 - y(t)) \\ \cdots \\ \dot{z}_n = z_{n+1} - \beta_n (z_1 - y(t)) + b_0 u \\ \dot{z}_{n+1} = -\beta_{n+1} (z_1 - y(t)) \end{cases}$$

$$z_1(t) \to y(t), z_2(t) \to \dot{y}(t), \cdots, z_n(t) \to y^{(n-1)}(t)$$

$$z_{n+1}(t) \to f(t, y, \dot{y}, \cdots, y^{(n-1)}, u, \dot{u}, \ldots u^{(n-1)}, w)$$

ω_o-optimization

- Observer bandwidth: ω_o

$$s^n + \beta_1 s^{n-1} + \ldots + \beta_{n-1} s + \beta_n = (s + \omega_o)^n$$

- Optimization:

  Maximize ω_o until noise reaches threshold

Control Law

$$u = -\frac{z_{n+1} + u_0}{b_0}$$

$$y^{(n)} = (f - z_{n+1}) + u_0 \approx u_0$$

$$u_0 = k_p(r - z_1) - k_{d_1} z_2 - \ldots - k_{d_{n-1}} z_n$$

$$s^n + k_{d_{n-1}} s^{n-1} + \ldots + k_{d_1} s + k_p = (s + \omega_c)^n$$

Design Procedure

- Step 1: Design a parameterized observer and feedback controller;
- Step 2: Design a transient profile with the equivalent bandwidth of ω_{ct};
- Step 3: Select an ω_o to 5-10 times larger than ω_{ct};
- Step 4: Set $\omega_c = \omega_o$ and increase both by the same amount until the noises levels and/or oscillations in the control signal and output exceed the tolerance;
- Step 5: Incrementally increase or decrease ω_c and ω_o individually, but keep the sum the same.

Example

$$\ddot{y} = (-1.41\dot{y} + 23.2T_d) + (23.2 - 40)u + 40u = f + 40u$$

$$\dot{z} = \begin{bmatrix} -3\omega_o & 1 & 0 \\ -3\omega_o^2 & 0 & 1 \\ -\omega_o^3 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 3\omega_o \\ 40 & 3\omega_o^2 \\ 0 & \omega_o^3 \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix}$$

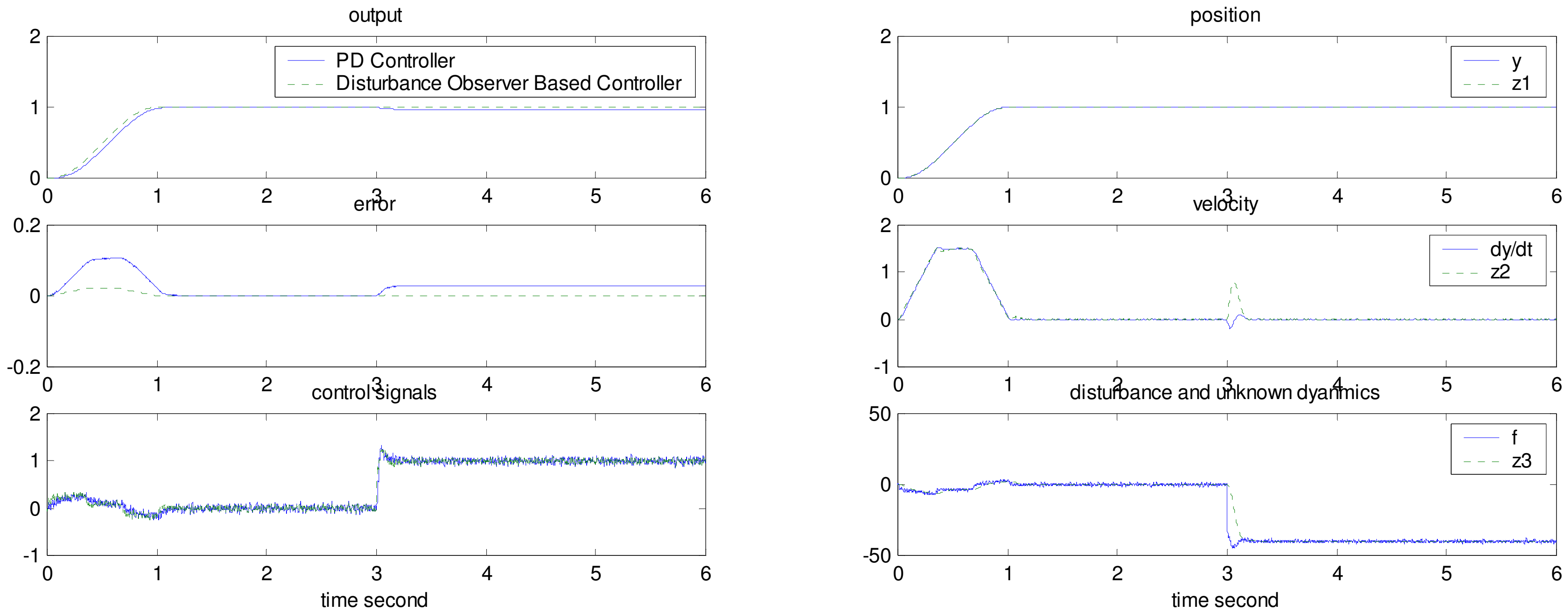
$$z_1 \to y, z_2 \to \dot{y}, and$$

$$z_3 \to f = -1.41\dot{y} + 23.2T_d + (23.2\text{-}40)\text{u}, \; as \; t \to \infty$$

$$u = \frac{u_0 - z_3}{40} \qquad u_0 = k_p(r - z_1) - k_d z_2$$

$$k_d = 2\xi\omega_c, \;\; \xi = 1, \text{ and } \; k_p = \omega_c^2$$

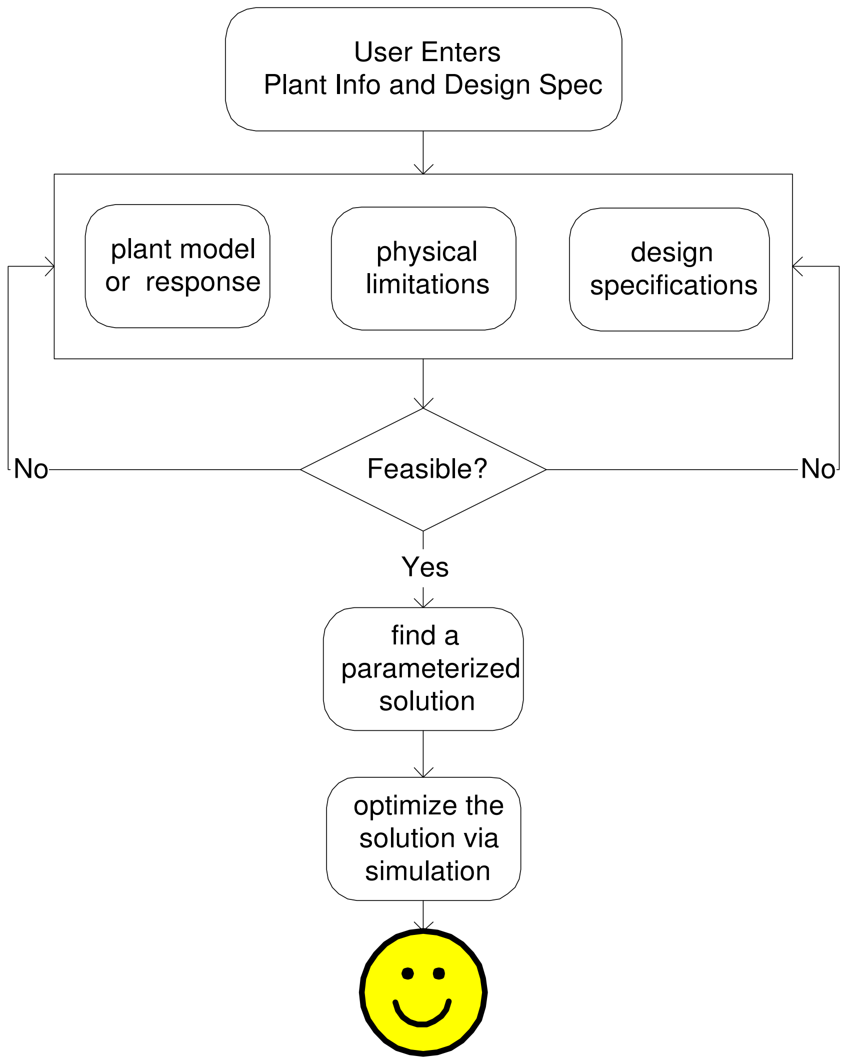
Simulation


output
PD Controller
Disturbance Observer Based Controller
error
control signals
time second
position
y
z1
velocity
dy/dt
z2
disturbance and unknown dynamics
f
z3
time second

Self-Tuning

- ω_o
  Automatically response to the noise level
- ω_c
  Bandwidth scheduling
- Scaling Factors (k,ω_p)
  Adaptation to plant dynamics variations

CAD Package Blueprint



Summary

- Three new concepts
  - Scaling
  - Parameterization
  - Practical Optimization
- Applications
  - From art to science
  - Stops reinventing the wheels
  - Optimization in real world