**Cleveland State University**
**Department of Electrical and Computer Engineering**
**Control Systems Laboratory**

# Experiment #2
# First Principles Modeling of the Controlled System (Torsion Mechanism)

## INTRODUCTION

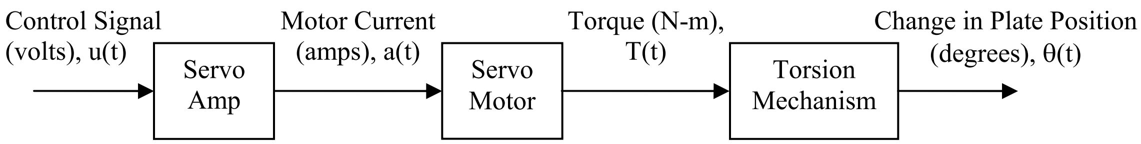
The successful design of a control system often requires the designer to develop a mathematical model capable of predicting the dynamic behavior of the system to be controlled so that that system's response to control actions can be understood and used in the process of designing the controller. The objective of this experiment is to demonstrate the use of the First Principles Method for developing a mathematical model of the dynamic behavior of the system to be controlled (in this lab it is the torsion mechanism).

What you will do here to achieve this objective is to (i) use theory to develop the form of a mathematical model of the torsion mechanism using the First Principles Method, (ii) make tests on the mechanism to determine values for the parameters in the model, and then (iii) make final adjustments to the model's parameters so that its response (observed using simulation) matches as closely as possible the response of the actual mechanism.

**Torsion Mechanism:** The torsion mechanism whose dynamic behavior you are to model was studied in Experiment #1. In that experiment you actually controlled the mechanism's behavior (change in plate position) in a closed loop fashion using a simple proportional controller; in this experiment you will build a model of the mechanism so that you can design more complicated closed loop controllers for it in later experiments.

The torsion mechanism is shown in Figure 1 and consists of a single inertia system (plate and two masses) rotated by a torque applied to its drive shaft by a servomotor. Friction and windage associated with the plate's movement manifest themselves in the form of a component of the load torque proportional to angular velocity. The servomotor is a brushless DC Servomotor connected to the plate's shaft through a rigid-belt/pulley system with a 3:1 speed reduction, and the motor itself—which has a torque constant (gain) of 0.086 Nm/A and negligible dynamics compared to the those of the torsion mechanism—is driven by a power amplifier which also has negligible dynamics compared to the torsion mechanism, and a gain of 1.5 A/V. The goal of this experiment is to come up with a model for the entire controlled system shown in Figure 1, i.e. a mathematical description of the dynamic behavior of the system whose input is the control signal, u(t), and whose output is the change in plate position, θ(t).

**Figure 1: <u>Torsion Mechanism to be Modeled</u>**




**<u>Plate Inertia</u>:** A top view of the plate with its two brass cylinders is shown in Figure 2. Each brass cylinder (including nut and bolt) has a mass of 0.5 Kg and a diameter of 5.0 cm; the plate itself has an inertia of 0.0019 Kg-m$^2$ and the motor and drive have a combined inertia (all reflected to the plate side of the belt/pulley system) of 0.0005 Kg-m$^2$. The cylinders are mounted so that their centers are each 4.0 cm from the plate's axis of rotation.

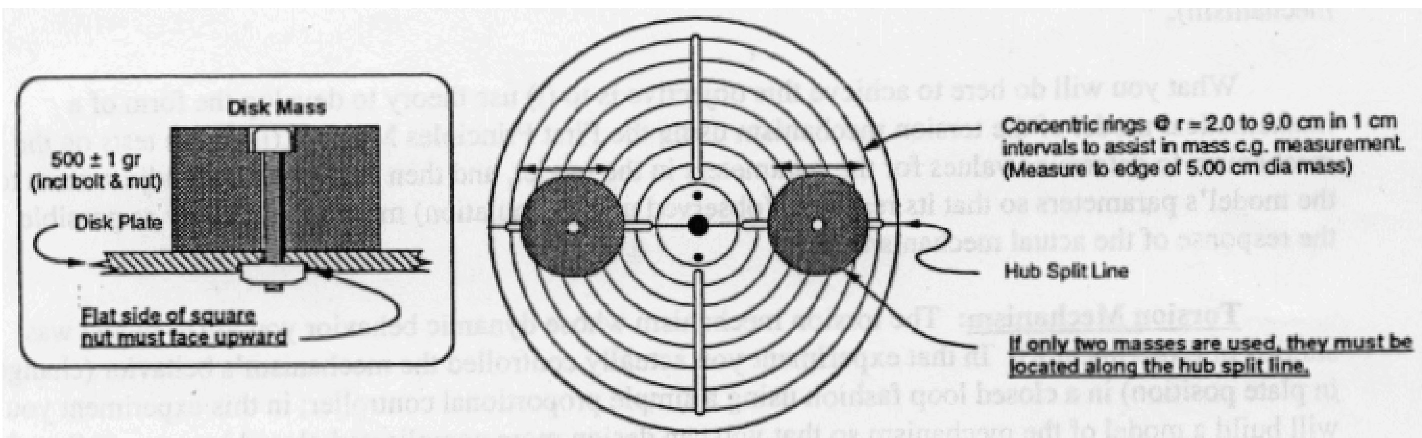



**Figure 2: <u>Plate and Masses</u>**

**<u>First Principles Method of</u> <u>Modeling</u> <u>Dynamic Systems</u>:** This method is based on using the physical laws—First Principles—that govern the dynamic behavior of a system to guide you in writing equations whose solutions will give you that dynamic behavior. You have done this many, many times before for electrical systems—now you will do it for an electromechanical system. The First Principles you used for electrical systems are Kirchhoff's Laws, those for mechanical systems are Newton's Laws, and an electromechanical systems uses both. The electrical part of your electromechanical system is so simple you will only have to use Newton's Law for the mechanical part. In fact, by this time in the lecture course you have already covered all of this material.

**Please note** that although the application of First Principles gives you the ***form*** of the equations governing the behavior of a system, it doesn't give you values for the ***parameters*** in the equations (resistor, inductor, capacitor values, mass and inertia values, pipe sizes - whatever the equations involve)—these have to be determined from data sheets and/or experimentally.

## PRE-LAB

1. Use the Parallel Axis Theorem to derive the total load inertia—plate, cylinders, and motor/drive inertia.

2. Use Newton's Law for rotational systems to derive the differential equation for the torsion mechanism whose input and output are motor torque, T(t), and change in plate position, θ(t), respectively. Use the symbol J for inertia, and use B (whose value will be determined experimentally later on) for the coefficient in the windage/friction term in the equation that represents that component of load torque proportional to angular velocity dθ(t)/dt. Remember that the belt/pulley system between the servomotor and the torsion mechanism produces a 3:1 speed reduction and therefore a 1:3 torque increase (a belt/pulley system is like a transformer—it has a high efficiency—and mechanical power is torque x angular speed (check out the dimensions), so the torque/speed product out of the belt/pulley system is about equal to the torque/speed product in). Assume the torsion mechanism's belt/pulley system has an efficiency of 100%.

3. Use the information given earlier for the servomotor and the servo amp to derive an overall differential equation for the entire controlled system whose input and output are the control signal, u(t), and the change in plate position, θ(t), respectively (see Figure 1).

4. Derive a transfer function for the entire controlled system from the differential equation for it that you derived above.

## PROCEDURE

**MAKE SURE ALL POWER IS TURNED OFF.**

**Part I—Determination of a Value for B:** The Pre-Lab work determined the controlled system's transfer function and values for all of its parameters except B; in this part of the experiment you will determine a value for B. The key to the method for doing this is to recognize two things: first, angular speed in radians/sec is equal to dθ(t)/dt, the term in the model involving B, and second, if the load is run at a constant speed, the angular acceleration $d^2\theta(t)/dt^2$ is zero. From inspection of the model you developed it should be clear that if you run the load at a constant speed and then measure the speed and the control signal used, you should be able to calculate a value for B. Remember: speed in the model is in rad/sec, but measured speed is in rpm.

There are different ways to run the load at different speeds; probably the best way—in order to keep the speed from drifting—is to use a speed control loop in which load speed is fed back and compared to a speed set point in order to generate the control signal that runs the motor. Devise an experimental set-up that will do this, and when you think you have it right call the instructor over to discuss your set-up. **DO NOT PROCEED FURTHER UNTIL YOU HAVE DISCUSSED YOUR SET-UP WITH THE INSTRUCTOR.**

**You should run the load in real time and at speeds in a speed range from 100 rpm to 400 rpm, and you should get values for B at four of five different speeds to see if there are differences. If the B values turn out to be different, a good final value for B would be the average of the values you obtained.**

## DISCUSSION QUESTIONS

1. Using a rotational mechanical system with one inertia, one damper, and an applied torque T(t), ***derive***—using Newton's Law for rotational mechanical systems—your differential equation model of the controlled system. In this derivation, use J for inertia and B for the damping term, and develop the model in terms of $\theta(t)$ and its derivatives, and the control signal u(t). (All this was done for the pre-lab, but should be included here as well.) You must ***derive*** the differential equation by applying Newton's Law—you can't just write the differential equation down next to the diagram.

2. Show your complete calculation of the value for the system inertia J. Make ***sure*** you include its units.

3. If you obtained different values for B at different speeds, ***discuss,*** first, why this might be so, and then ***discuss*** the significance of using one value for B in the model—or, equivalently, the significance of using one model for a range of speeds.

4. Write down the value of B you are proposing for use in the model. Make ***sure*** you include its units.

5. In question 1 above you derived a differential equation model of the controlled system. In this question you are asked to use that differential equation to come up with a transfer function model of the controlled system. Write down the transfer function model of the controlled system (input is u(t) and output is $\theta(t)$) showing explicitly where in it J, B, and any constant term K appear, and then—right next to the model—***write down your values for J, B and K.***

6. Now write down the transfer function model in the standard form $\theta(s)/U(s) = K_1/s(\tau s + 1)$. ***Make sure*** you show the relationships between J, B, K, $K_1$ and $\tau$.

**<u>Part II—Adjustments to the Model's Parameter Values</u>: A** model is only as good as its ability to predict the actual response of the system it is a model of. In this part of the experiment you will compare your model's response to that of the actual mechanism, and then make adjustments to its parameter values so that its response matches as closely as possible the mechanism's response. This is called "tuning" the model to the system it is modeling. To do the tuning process one typically compares the step responses of the model and the real system, and then adjusts the model's parameters to match its step response to the real system's step response.

a) To simplify the tuning process with this equipment, we will work with the model's ability to predict the speed response of the mechanism rather than its position response. Noting that angular speed $\omega(t) = d\theta(t)/dt$ (and therefore that angular acceleration $d\omega(t)/dt = d^2\theta(t)/dt^2$), the model you developed can be written in terms of $\omega(t)$ and its derivatives rather than in terms of $\theta(t)$ and its derivatives, i.e. the output of the model − or the model's response – can be taken to be speed rather than position. The model's parameters can then be adjusted by comparing the model's and the mechanism's speed responses rather than their position responses—and this is a benefit with this equipment since the mechanism's position response to a step is a ramp, and it is hard to make parameter value adjustments when working with ramps. **Rewrite the model so that the control signal, u(t), is its input and load speed, $\omega(t)$, is its output, i.e. the model will involve $\omega(t)$ and its derivatives rather than $\theta(t)$ and its derivatives. Once you have done this, derive the model's transfer function between U(s) and $\theta(s)$. WHEN YOU HAVE THE TRANSFER FUNCTION SHOW IT TO THE INSTRUCTOR BEFORE PROCEEDING TO MAKE SURE YOU ARE WORKING WITH THE CORRECT MODEL.**

b) Develop a real-time experimental set-up that allows you to do the following three things: (i)step the mechanism's load speed from 0 rpm to about 300rpm (i.e., make step changes in the *control signal* so that the load speed swings between these values), (ii) apply the same control signal to your model of the mechanism, and (iii) record on the same graph (using a 2-input MUX block from the "Connections" library in Simulink as the graph's input) the speed response of the model and the speed response of the mechanism. **DO NOT TURN ON THE POWER. WHEN YOU THINK YOU HAVE A GOOD SET-UP, DISCUSS IT WITH THE INSTRUCTOR.**

c) Now make adjustments to the model's parameters so that the model's response matches as closely as possible the mechanism's response. You might start by adjusting B, but in the end all parameters in the model should be adjusted to get the best fit. Make recordings of the initial responses as well as one or two intermediate response, and then the final responses. **WHEN YOU THINK YOU HAVE A GOOD FIT, DISCUSS IT WITH THE INSTRUCTOR.**

## DISCUSSION QUESTIONS

1. ***Discuss*** your model tuning experience. What parameters affected what part of the response?

2. Make a table that shows the values for J, B, K, $K_1$, and $\tau$ that you determined in Part I (based on that part's combination of calculation, experimentation, and the values you were given (obtained from spec-sheets) for servo amp gain, motor torque constant, and pulley ratio) and those that you obtained in Part II based on tuning the model.

3. ***Discuss*** the differences and similarities of the values in the table that you just prepared above in answer the previous question. How do you account for the differences? It is not enough to just ***describe*** the differences and similarities—you must discuss these things and attempt to give reasons for the differences.